Human Capital, Capital Markets and Earnings
Uncertainty

Assume a two-period, one-person model. The person has a time endowment of $T$ in period 1 which can be allocated to schooling, $s_1$, or to work, $T - s_1$, for a real wage rate $w_1$. The person’s hours of work are fixed in period 2 but total real earnings, $e_2$, depend on schooling in the following ways:

\[
e_2(0) = 0 \quad \quad e'_2(s_1) > 0 \quad \quad e''_2(s_1) < 0.
\]

Denote the person’s lifetime consumption plan by $(c_1, c_2)$ and suppose the person’s preferences can be represented by $U(c_1, c_2)$.

**Perfect capital markets**

Assume the person has access to a perfect capital market with real interest rate $r$. Then the person’s lifetime budget constraint is

\[
c_1 + \frac{c_2}{1 + r} = w_1 (T - s_1) + \frac{e_2(s_1)}{1 + r}.
\]

In this setting the person could optimize by picking $s_1$ to maximize the right-hand-side of the budget constraint and then, given the optimal schooling value of $s_1^*$, pick $(c_1, c_2)$ to maximize $U(c_1, c_2)$. Equivalently, the person could solve

\[
\text{Max}_{c_1, s_1} \quad U(c_1, (1 + r) (w_1 (T - s_1) - c_1) + e_2(s_1)).
\]

The first-order conditions are

\[
U_1 + U_2 (1 + r) (-1) = 0 \quad \quad U_2 [(1 + r) w_1 (-1) + e'_2(s_1)] = 0.
\]

These equations can be written as

\[
\frac{e'_2(s_1)}{w_1} = 1 + r = \frac{U_1}{U_2}.
\]
The second equation is the Econ 290 MRS condition — in a two-period model, in equilibrium, the person equates the MRS between $c_1$ and $c_2$ to $1 + r$. The first equation says that at the optimal level of schooling one plus the marginal return to further education should equal one plus the real rate of interest.

**Imperfect capital markets**

Now assume that capital markets are imperfect in the sense that the person cannot borrow against second-period earnings. If the person doesn’t want to borrow against second-period earnings, that is, $w_1(T - s_1^*) - c_1^* \geq 0$, then this imperfection in the capital market doesn’t matter. If $w_1(T - s_1^*) - c_1^* < 0$ then this capital market imperfection will matter. The separability between the schooling decision and the consumption allocation decision breaks down and the budget constraints become the following two equations.

\[
\begin{align*}
    c_1 &= w_1(T - s_1) \\
    c_2 &= e_2(s_1),
\end{align*}
\]

so the person’s optimization problem can be written as

\[
\max_{s_1} U(w_1(T - s_1), e_2(s_1)).
\]

The first-order condition is

\[
U_1 w_1 (-1) + U_2 e_2'(s_1) = 0 \text{ or } \frac{e_2'(s_1)}{w_1} = \frac{U_1}{U_2} > 1 + r.
\]

Here the chosen level of $s_1$ lies below $s_1^*$, the chosen level of $c_1$ will be less than with perfect capital markets, and utility will be lower than it would be with perfect capital markets. The competitive equilibrium is not Pareto efficient and some would argue this model makes a case for government subsidization of education.

**Imperfect capital markets and earnings uncertainty**

Let the person’s utility as a function of first and second-period consumption be $u(c_1) + v(c_2)$; assume $u$ and $v$ are strictly increasing, strictly concave and have positive third derivatives. Further assume she is unable to borrow against second-period earnings to finance first-period consumption, and, given her preferences, this constraint binds. In addition, suppose that for some $\delta > 0$, the person’s second-period earnings are $e_2(s_1) + \delta$ with probability $1/2$ and $e_2(s_1) - \delta$ with probability $1/2$. Assume the person acts to maximize expected utility. Does increasing earnings uncertainty, that is increasing $\delta$, move $s_1$ towards its socially-efficient level, that is, increase it?

We have
\[ \text{EU} (s_1, \delta) = u(c_1) + \frac{1}{2} v(c_2^+) + \frac{1}{2} v(c_2^-) \]
\[ = u (w_1(T - s_1)) + \frac{1}{2} v(e_2(s_1) + \delta) + \frac{1}{2} v(e_2(s_1) - \delta) \]

By the implicit function theorem
\[ \text{Sign} \frac{ds_1}{d\delta} = \text{Sign} \text{EU}_{12}(s_1, \delta). \]

Then
\[ \text{EU}_1(s_1, \delta) = -w_1 u' (w_1(T - s_1)) + \frac{1}{2} e'_2(s_1) (v' (e_2(s_1) + \delta) + v' (e_2(s_1) - \delta)) \]
\[ \text{EU}_{12}(s_1, \delta) = \frac{1}{2} e'_2(s_1) (v'' (e_2(s_1) + \delta) - v'' (e_2(s_1) - \delta)) \]
\[ = \frac{1}{2} e'_2(s_1) (v'' (c_2^+) - v'' (c_2^-)) \]

Since we have assumed \( v''' > 0 \), which means \( dv''(c)/dc > 0 \), and \( c_2^+ > c_2^- \), the right-hand side of the last equation must be positive; here \( ds_1/d\delta > 0 \). The result says that in a world with imperfect capital markets, where the person may choose less than the socially-efficient level of schooling, increasing earnings uncertainty helps to increase efficiency.

This example can be used to make an important and quite general point. In this setting, introducing one source of market failure leads to a private equilibrium that is Pareto inefficient. One might think that introducing uncertainty on top of the capital market imperfection, might cause the private equilibrium to deviate even further from efficiency. This is not true in this and many other settings. At some point (not for this course) you should read: