SOCIAL SECURITY AND SAVINGS PLANS IN OVERLAPPING-GENERATIONS MODELS

John B. BURBIDGE
McMaster University, Hamilton, Ontario L8S 4M4, Canada

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This paper examines the impact of social security and saving plans on a simple overlapping-generations model with no uncertainty. The model is related to, and provides a reconciliation of, the Diamond and Samuelson-Gale models of public debt and public capital. Savings plans and pay-as-you-go social security are seen to have quite different effects depending on the assumptions governing the growth rates of public debt and capital, and it is also shown that the social security program proposed by Feldstein is infeasible in the steady state he adopts as a starting-point.

1. Introduction

There has been, and continues to be, great interest in the effects of changes in social security programs on the capital stock and on the welfare of successive generations. Given the magnitude of the tax revenues and benefits involved, it is clear that a general-equilibrium model should be used. One such framework for studying social security is the general-equilibrium, overlapping-generations model originally presented by Samuelson (1958), and developed further by, amongst others, Diamond (1965), Aaron (1966), Stein (1969) and Gale (1972, 1973, 1976). Although many have used this framework to study social security and savings plans, they have not focused on the kinds of equilibria that can exist, or their stability properties. The main purpose of this paper is to do this in a very simple overlapping-generations model with no uncertainty.

Social security and savings plans involve public debt and capital. To understand how public debt and capital work in overlapping-generations models, it is instructive to ask why, given tastes and technology, there are...
two steady states in Samuelson (1958) but only one in Diamond (1965). Gale (1972, 1973, 1976) has argued that the two equilibria of the Samuelson model are distinguished by the relationship between private wealth and the stock of capital. In the balanced equilibrium, akin to the no-debt equilibrium in Diamond (1965), private wealth equals the stock of capital; in the other, a golden-rule equilibrium, the rate of interest equals the rate of growth of real output and private wealth may be greater than, or less than, the total stock of capital. In this steady state there is, in effect, an institution, which could be the government, that owns some of the capital stock or holds debt that corresponds to private wealth. Under Diamond’s assumptions the balanced equilibrium is stable, but under Gale’s it may not be, and the main purpose of section 2 is to show that the critical assumptions that give rise to the different conclusions are those that govern the growth rates of public debt and capital. This is also a theme of the next three sections, in which I analyze the effects of introducing a savings plan, pay-as-you-go social security and the social-security program discussed by Feldstein (1974a, 1977a). Savings plans and pay-as-you-go social security are seen to have different effects on the steady-state capital–labour ratio, depending upon the assumptions made. My analysis also reveals that Feldstein’s social-security program is infeasible in the steady state that he takes as a starting point.

2. Public debt and capital

2.1. The model

In this section I outline a simple overlapping-generations model that contains the essentials of both Diamond (1965) and Gale’s work extending Samuelson’s consumption-loans model. Gale (1973), following Samuelson, assumed a pure-exchange economy where endowments are not storable, while Diamond adopted the framework of a neoclassical economy with production and storable output. Nothing essential hinges on this difference; the main properties of the Samuelson–Gale model carry over to an economy with production. I assume there is only one good, corn, which is produced each period with a constant-returns-to-scale technology, employing labour and the stock of corn available at the beginning of the period. I write the production function as:

\[ y_t = f(k_t), \quad f' > 0, \quad f'' < 0, \]  

where \( y_t \) is output per unit of labour, and \( k_t \) is capital per unit of labour, both at time \( t \). I assume all markets are perfectly competitive so that each

\[ ^2 \] I have abstracted from technical progress to simplify the exposition. The main results of the paper hold when technical progress is admitted.
factor is paid its marginal product. Letting $w_t$ and $r_t$ be the real-wage rate and the interest rate, at time $t$:

\begin{equation}
    r_t = f'(k_t)
\end{equation}

and

\begin{equation}
    w_t = f(k_t) - k_t f'(k_t).
\end{equation}

I assume that all persons born at time $t$ are identical; each lives for two periods, works full time in the first and not at all in the second, and leaves no bequest.\(^3\) I assume that there is an institution that may own capital or may borrow from private individuals; it reinvests a fraction $s_t$ of the interest payments on whatever capital it owns. In Diamond (1965), the institution is the government and therefore has coercive powers; I assume it takes from the young, in taxes, a proportion $\theta_t$ of their wages each period, promising nothing in return for these taxes. In Gale's work, the institution has no such power and $\theta_t$ is always zero. To simplify, for the moment, I abstract from questions dealing with the origin of the capital or debt. I assume the population grows at a fixed rate $n$, and that $(1+n)^t$ is the number of young people born in period $t$. $a_t$ denotes the institutional capital per young person in period $t$, and is measured in corn at the beginning of period $t$. $e_t$ is the corresponding institutional debt. Under these assumptions:

\begin{equation}
    (1+n)a_{t+1} = (1+r_s) a_t
\end{equation}

and

\begin{equation}
    (1+n)e_{t+1} = (1+r_t)e_t - \theta_t w_t.
\end{equation}

Let each private individual consume $c_0^t$ and $c_1^t$ in the two periods of his or her life\(^4\) and let $h_t$ be total private wealth at the beginning of period $t$, divided by the number of people born at $t$. Since private wealth equals the saving of those who were young in the previous period,

\begin{equation}
    h_t = \frac{(1-\theta_{t-1})w_{t-1} - c_{t-1}^0}{1+n}.
\end{equation}

Given the no-bequest assumption:

\begin{equation}
    c_{t-1}^1 = (1+n)(1+r_t) h_t.
\end{equation}

\(^3\)I discuss some of the effects of modifying the labour supply assumptions below. Admitting bequests and gifts to the model produces a third type of overlapping-generations model, the Barro model, which is very similar to optimal growth models [see Burbidge (1983)].

\(^4\)Note that the $t$-subscripts in the consumption variables stand for a person's generation, not the time period.
I assume each young person maximizes a well-behaved utility function, $u = u(c^0_t, c^1_t)$, which does not change from one generation to the next. The consumption of each young person can be written as:

$$c^0_t = g(k_t, k_{t+1}, \theta_t).$$  \hspace{1cm} (8)

To close the model I employ an asset identity. This identity is a generalization of similar identities in Diamond (1965) and Gale (1973), and it permits a simple and intuitive understanding of different overlapping-generations models. It states that, with production, private assets plus net institutional assets equals the stock of capital:

$$h_t + a_t - e_t = k_t.$$  \hspace{1cm} (9)

In the Samuelson-Gale model, where output is nonstorable, the right-hand side would equal zero. This identity, the individual's budget constraint, and eqs. (1)-(7) imply eq. (10), the GNE identity:

$$y_t = c^0_t + c^1_{t-1} + (1 + n)k_{t+1} - k_t + (1 - s_t)r_ta_t.$$  \hspace{1cm} (10)

For both Diamond and Samuelson-Gale $y_t, w, r, c^0_t, c^1_{t-1}, h_t,$ and $k_{t+1}$ are endogenous variables at time $t$. Diamond (1965) focused on the case where $a_t$ was set equal to zero, $e_{t+1}$ was set equal to a constant, $e$, and eqs. (1)-(3) and (5)-(9) determined the seven endogenous variables plus $\theta_t$. More generally, the Diamond kind of model is one in which $a_{t+1}$ and $e_{t+1}$ are fixed by government policy and $\theta_t$ and $s_t$ are the extra endogenous variables determined by a system of nine equations. Samuelson and Gale, who were much less specific about the nature of the institution, implicitly reversed this, taking $s_t$ and $\theta_t$ to be exogenous and $a_{t+1}$ and $e_{t+1}$ to be endogenous. In fact, they (implicitly) assumed that $s_t$ equals unity and $\theta_t$ equals zero. The two models yield different steady states, and it is to these, and their stability properties, that I now turn my attention.

\section*{2.2. Steady states and their stability properties}

A steady state is a situation where $k_t$, and $e_t$ or $a_t$, are unchanging over time. Dropping time subscripts to indicate steady-state values, eqs. (5), (6), (8) and (9) imply:

$$w - (r - n)e - g \left( k, k, \frac{(r - n)e}{w} \right) - (1 + n)k = (1 + n)(e - a).$$  \hspace{1cm} (11)
Given eqs. (2) and (3), and with $a$ and $e$ exogenous, (11) is a nonlinear equation in $k$. To proceed further, I follow the analysis of internal debt in Diamond (1965) and assume that (11) has a unique solution and that the steady state, which it represents, is stable.\(^5\) Eq. (9), and the assumption of stability, which is that $dk_i/dk_{i-1} < 1$, imply:

$$dh_i/dk_{i-1} < 1.$$  \hspace{1cm} (12)

Now consider the Samuelson–Gale model. Here, $a$ and $e$ are endogenous, $s$ is fixed and $\theta$ equals zero. Eqs. (4), (5) and (9) then imply that, for given tastes and technology, there are two kinds of steady states: type A where $rs = n$, with $a > 0$, or $r = n$ (the golden rule), with $e > 0$, and type B where $a = e = 0$, so that $h = k$ (this is a balanced steady state where private wealth equals the stock of corn).

Let ‘A’ subscripts denote variables in a Gale type-A equilibrium and ‘B’ subscripts variables in his type-B equilibrium. Barring the case where the two types of equilibria coincide, tastes and technology must be such that $r_B > n$ or $r_B < n$. Gale has named the first the ‘classical’ case after Fisher’s (1930) theory of interest and the second the ‘Samuelson’ case because it corresponds to the setting in Samuelson’s article on consumption loans. I now state and prove the following result, which is a production-economy analogue of theorem 2 in Gale (1973).

**Result 1.** The model is classical (Samuelson) if and only if private wealth is less than (greater than) the stock of capital in the type-A steady state.

**Proof.** I need to show that $(r_B - n)(h_A - k_A) < 0$. Consider a classical model in a B steady state. I know $r_B > n$, $h_B = k_B$ and by (12) that there is a neighbourhood of the type-B equilibrium in which $dh/dk < 1$. A small increase in $k$, which is a step towards the type-A equilibrium, therefore implies that $h - k < 0$. Since there is only one type-B equilibrium, where $h_B - k_B = 0$, $h_A - k_A < 0$. The argument for the Samuelson case is similar, except that $r_B < n$.

Result 1, in conjunction with eqs. (4), (5) and (9), yield the stability analysis of the Samuelson–Gale steady states. This is summarized in result 2.

**Result 2.** In a classical (Samuelson) model, the type-A equilibrium is stable (unstable) and the type-B equilibrium is unstable (stable).

\(^5\)In fairly extensive simulation work with two-period models I have never found this to be a problem. Recent work by Kehoe and Levine (1982) suggests that problems of relative-price indeterminacy do arise, however, in multi-period models.
A sketch of a proof for a classical model should provide the reader with an intuitive understanding of this result and of the functioning of the Samuelson–Gale model. Consider a classical type-B equilibrium. The creation of an infinitesimal amount of public capital will increase the capital–labour ratio, pushing the economy towards the type-A equilibrium (by result 1). According to eq. (4) this institutional (or government) asset will grow at \( r_s \). Provided \( r_{B_0} > n \), this capital will grow more quickly than private wealth until the new type-A equilibrium is attained, where \( r = r_A = \frac{n}{s} \). If debt had been created it would [by (5)] have grown at \( r \) and, mechanically speaking, the economy eventually would have tried to consume more than its output plus its existing capital stock. In a perfect-foresight model, such as this one, it is better to say that debt could not be introduced into a classical type-B equilibrium under the Samuelson–Gale assumptions. In a classical type-A equilibrium everything revolves around the equality between the (after-tax) interest rate and \( \frac{n}{s} \) [eq. (4)]. Barring interest income taxes, no (small) perturbation could affect the steady-state capital–labour ratio.

![Fig. 1. Possible A and B equilibria.](image)

It may be helpful to represent these results diagrammatically. In a type-A equilibrium, \( y - nk \) is maximized. In fig. 1, \( OF \) equals the highest level of \( y - nk \) possible, \( r \) equals \( n \), and the slope of \( FM \) equals \( 1 + n \) in absolute value. \( I_1 \) is the highest level of utility an individual can attain in a steady state in this economy and in this sense point \( A \) in fig. 1 is Pareto optimal. In a Samuelson–Gale type-B equilibrium, either \( n \) exceeds \( r_B \) (Samuelson) or \( r_B \) exceeds \( n \) (classical). In either case, \( y - nk \) must be lower and thus in per capita terms the economy is stationed on a budget constraint like \( F'M' \), which again has a slope of \( 1 + n \). The individual will equate \( MRS_{c0_1} \) to

\[ c^0 + c^1/(1 + n) \]

\(^6\)The first-order condition for maximizing \( y - nk \) is \( r = n \). By the steady-state version of eq. (10), maximizing \( y - nk \) means maximizing the present value of consumption, \( c^0 + c^1/(1 + n) \).
1 + r_B and hence a type-B equilibrium will occur either at a point like B_1 (where n > r_B) or B_2 (r_B > n). If it is B_2, then the capital–labour ratio at B_2 will be less than k_A and I have shown that in a Gale model the phase line is something like the solid curve in fig. 2. If the economy is at a point like B_1 in fig. 1, then the corresponding phase line in fig. 2 is like the dotted curve.

Fig. 2. Stability of A and B equilibria.

Researchers in public finance often base their analyses on steady-state equilibria in which the interest rate exceeds the population growth rate [e.g. Summers (1981)]. The superficial similarity between the steady state of the Diamond model and the classical B-equilibrium of the Samuelson–Gale model hides a fundamental difference, which is that if the Diamond equilibrium is stable, the Samuelson–Gale is not. I now turn to an examination of some social security and savings programs that illustrate this point.

3. A public savings program

Feldstein (1974b, 1977b) suggested that the U.S. Government undertake a public savings program to raise the capital stock towards the golden-rule level. Specifically, he argued that the social-security tax rate should be raised to finance the purchase of government debt or physical capital, and also that the interest earned on the fund should be reinvested [Feldstein (1977b, pp. 45–46)]. In his paper on ‘The optimal financing of social security’ (1974b), he presented a sketch of a partial-equilibrium life-cycle model and concluded that a substantial payroll tax increase would be required to push the economy to the golden rule.

The situation Feldstein had in mind is one where the economy is initially in a steady state with a capital–labour ratio that is less the the golden-rule
level. In these circumstances the two Samuelson–Gale equilibria would be the initial type-B equilibrium and a type-A equilibrium determined by $rs = n$ [eq. (4)]. If the government taxes the young just in period 0 (at $\psi_0$) to create $a_1$ ($=\psi_0 w_0/(1+n)$) and saves a fraction $s$ of the interest on this capital, the analysis of section 2 shows that the initial type-B equilibrium is unstable, provided, of course, that the type-A equilibrium has a higher capital stock than the type-B equilibrium (i.e. $s > n/r_B$). This one-shot payroll tax would lead eventually to the golden-rule steady state if $s$ were equal to unity. Alternatively, if the government had some fixed target for $a_t$, which once attained would be maintained, then the economy would behave like the Diamond model. The one-shot payroll tax would push $a_t$, for $t \geq 1$, to a fixed level above $a_0$, equal to $a_1$, and the economy would move to a higher capital stock. In table 1 I illustrate the effects of a one-shot payroll tax to finance government capital under the two different postulates about government target variables. To carry out these simulations I have employed a CES production function:

$$y_t = (\beta + (1-\beta)k_t^\rho)^{1/\rho}, \quad \rho < 1, \quad \sigma = 1/1-\rho,$$  

(13)

Table 1

<table>
<thead>
<tr>
<th>Time</th>
<th>$k_t$</th>
<th>$a_t$</th>
<th>$u_t$</th>
<th>$k_t$</th>
<th>$s_t$</th>
<th>$u_t$</th>
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<td>0.442430</td>
<td>0.085989</td>
<td>0.604978</td>
<td>0.444460</td>
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<td>0.087524</td>
<td>0.612498</td>
<td>0.452488</td>
</tr>
</tbody>
</table>

$s = 0.9 \quad a_{-1} = a_0 = 0; \quad a_t = 0.001695, \ t \geq 1$

$\rho = 0.01, \ y = 0.01, \ \beta = 0.7, \ \delta = 0. \quad n = 1, \ \psi_0 = 0.01$

Notes: 1. $k_t$ and $a_t$ are the total capital and government capital–labour ratios at the beginning of period $t$; $u_t$ is the utility level of each individual in the generation born at time $t$.
2. $s_t$ is determined in a Diamond model only when $a_t > 0$ [eq. (4)].
3. If a generation is taken to be 30 years, $n = 1$ implies an annual population growth rate of $\ln 2/30$ or 2.31 percent.
where $\sigma$ is the elasticity of substitution, and a constant relative risk aversion utility function:

$$u_t = \frac{(c_t^0)^{\gamma}}{\gamma} + \frac{1}{1+\delta} \frac{(c_t^1)^{\gamma}}{\gamma}, \quad \gamma < 1.$$ (14)

In the table, $a_1$ is the same for both models but $a_2$ will be higher in a Samuelson–Gale model than in the Diamond model because $sr_1$ exceeds $n$ in the former and equals $n$ in the latter. Higher values of $k_t$, which imply lower levels for $r$, initially depress the consumption of the old and utility levels in a Samuelson–Gale model relative to a Diamond model, but this effect on utility is eventually reversed by the higher level of $w_t$ in the former. The $u_t$ columns show an increase in welfare looking across steady states, but this comes at the expense of the generation that is young when the plan is introduced and one or two of its successors.

4. Pay-as-you-go social security

A pay-as-you-go (or just pay-go) social security program is defined to be a program in which social security tax revenues equal benefits in each period. Letting $\theta_t$ be the tax rate and $b_t$ the per capita benefits paid to the old, both at time $t$, eq. (15) holds identically:

$$(1+n)^{t-1} b_t = (1+n)^t \theta_t w_t.$$ (15)

Consider the introduction of a pay-go plan in the context of the model in section 2. Assume there is no government debt initially ($e_{-1} = 0$). In period 0 the government sets $\theta_t$ at $\theta > 0$; it taxes $\theta w_o$ from the young and pays this to the old. $c_{-1}^1$ will equal $(1+n)(1+r_0)h_0 + (1+n)\theta w_0$. At the beginning of period 1 the government owes $b_1$, or, by (15), $(1+n)\theta w_1$ to those who are now old; its debt, $(1+n)e_1$, is $(1+n)\theta w_1/(1+r_1)$. Eq. (16) thus replaces eq. (5) for $t \geq 1$:

$$e_t = \frac{\theta w_t}{1+r_t}. \quad (16)$$

The only other modifications of the section 2 model also arise because the payment of taxes by the young entitles them to benefits when old, according to the pension formula expressed in eq. (15). Eqs. (6') and (8') replace their counterparts:

$$h_t = \frac{(1-\theta_{t-1})w_{t-1} - c_{t-1}^0}{1+n} \cdot e_{t-1}, \quad t \geq 1.$$ (6')
These changes to the model of section 2 bring the Diamond and Samuelson–Gale models closer together. Making \( e_t \) endogenous and \( \theta_t \) exogenous moves the Diamond model towards the Samuelson–Gale assumptions, and replacing (5) by (16) removes the source of instability in the Samuelson–Gale model. It can be shown, and table 2 illustrates, that the two models give the same equation for the short-run effect of increasing \( \theta \) on \( k_t \).

It is also true that, starting from steady states with no public debt or capital, a pay-go plan reduces the capital-labour ratio for both models. This accords with Kotlikoffs's (1979) numerical simulations of a pay-go plan and is naturally similar to Diamond's result for the effects of introducing internal debt (1965, p. 1142), but, as one can see from (16), pay-go social security and internal debt are not identical along the adjustment path.

**Table 2**  
Effects of pay-go social security starting from a steady state with government capital

<table>
<thead>
<tr>
<th>Time</th>
<th>Diamond model</th>
<th>Samuelson–Gale model</th>
</tr>
</thead>
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<td></td>
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<td>( a_t )</td>
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<td>0.776447</td>
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</table>

Here, the only difference between the two models arises when there is institutional capital in the initial steady state. In a Diamond model, institutional capital per young person, \( a_t \), would not change by assumption and steady-state \( k \) would fall. In a Samuelson–Gale model, steady-state \( k \) would be driven by equality between \( R \) and \( n/s \), which would be taken to be constant. The effects of these differences are illustrated in table 2.

The two models start from the same steady state. In both, the pay-go plan raises welfare for the two generations that are alive when it is introduced. A
government that concerned itself solely with the welfare of those alive at the moment would introduce pay-go social security. In a Diamond model, $s_t$ falls to maintain $a_t$ constant and $u_t$ decreases as the economy moves away from the golden-rule ($r = n$) steady state. In Samuelson–Gale, with $s_t$ fixed, $a_t$ must increase to bring $r_t$ back into equality with $n/s$. Unless $s$ equals unity ($r = n$), the pay-go plan reduces steady-state welfare, even though $k$ is unaffected, because the plan cuts the present value of lifetime wealth, $(1 - \theta)w + [(1 + n)\theta w/(1 + r)]$.

5. Feldstein's theoretical analysis of social security

If a 'fully funded' social security program is defined to be one where the present value of contributions equals the present value of benefits for each individual, then the program has no effect on any individual's budget constraint, and thus has no effect on the economy, either in the short run or the long run [see Diamond (1965, p. 1136)]. Moreover, this is true whether or not each individual can choose how much to work in each period. If a 'fully funded' program is defined to be one where the present value of each generation's tax payments equals the present value of its benefits in the aggregate, then there may be steady-state effects when an individual can vary his hours of work in a period in which he faces a payroll tax.

A program that appears to be much like a full-funded plan but, in fact, has very different effects is the one analyzed by Feldstein (1974a, 1977a). He assumed that government taxes the young in each period, promising to repay taxes and accumulated interest next period. However, instead of lending the tax revenue from the young to the firms, when the plan is first introduced the government gives it to the old who consume it entirely. How does this kind of social security fit into the section 2 model and what are its short-run and long-run equilibrium effects?

I represent this plan, as I did the pay-go, by assuming that there is a permanent increase in the payroll tax rate at time 0 and that $e_0$ is zero (there is no government debt before the plan is introduced). Now consider the evolution of $e_t$. $(1 + n)e_1$ is the amount owed to the old at the beginning of period 1, and it must equal $\theta w_0$, so that $e_1$ equals $\theta w_0/(1 + n)$. In period 1, the old are paid $(1 + r_1)\theta w_0$, and the young are taxed and owed $(1 + n)\theta w_1$. Thus, $(1 + n)^2 e_2$ equals $(1 + r_1)\theta w_0$. In general, then, $e_t$ can be written as:

$$e_t = (1 + r_{t-1}) \ldots (1 + r_1) e_1/(1 + n)^{t-1}, \quad t \geq 1.$$  

This equation is just (5) with the $\theta_t$ term omitted. The other modifications of

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1. I have focused on the 'classical' case where $r > n$ because most public-finance economists appear to think this is the realistic one. If $r = n$, the Diamond model shows that pay-go social security would raise steady-state welfare, and the welfare of each generation, along the adjustment path.

2. This point is taken up briefly at the end of this section.
the section 2 model are that the $\theta$ terms must be dropped from (6) and (8) and $c_{t-1}$ must be set equal to $(1 + r_p)(1 + n)k_o + \theta(1 + n)w_o$.

It is apparent from (17) that Feldstein’s social security program creates government debt that grows at $r$, and hence that the Samuelson–Gale model describes the effects of this program. In the classical, type-B steady state, which Feldstein (1974a, 1974b, 1976, 1977a, 1977b) and others [Kotlikoff (1979), Hu (1979)] take to be the relevant starting point, a mechanical implementation of Feldstein’s program would drive the capital stock to zero. An illustration of this process is provided on the left-hand side of table 3.

With perfect foresight, of course, as I stated in the analysis of Samuelson–Gale equilibria in section 2, people would know what was going to happen and would not permit the government to implement this particular social-security program. This program is, therefore, infeasible. The only classical steady state in which the program could exist is the golden rule, and then it can have no effect on the steady-state capital–labour ratio, which is given by the equality between $r$ and $n$. One can show that if individuals are permitted to vary their hours of work in both periods in this steady state, they will choose to work less with a higher payroll tax rate and thus the capital stock will be reduced.\(^9\) The effects of this kind of social security on a classical type-A equilibrium are also illustrated by a numerical example in table 3.

### Table 3

**Effects of Feldstein’s social security program.**

<table>
<thead>
<tr>
<th>Time</th>
<th>$k_t$</th>
<th>$e_t$</th>
<th>$u_t$</th>
<th>$k_t$</th>
<th>$e_t$</th>
<th>$a_t$</th>
<th>$u_t$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.448486</td>
<td>0.176782</td>
<td>0</td>
<td>0.071489</td>
<td>0.586320</td>
</tr>
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<td>0</td>
<td>0.071489</td>
<td>0.585886</td>
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<td>0.590448</td>
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<tr>
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<td>0.071489</td>
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</tr>
<tr>
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<td>0.441668</td>
<td>0.174607</td>
<td>0.002107</td>
<td>0.071787</td>
<td>0.582857</td>
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<tr>
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<td>0.003102</td>
<td>0.441503</td>
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<td>0.002116</td>
<td>0.072098</td>
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<tr>
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<td>0.444763</td>
<td>0.175482</td>
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<td>0.072595</td>
<td>0.584141</td>
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</tr>
<tr>
<td>11</td>
<td>&lt;0</td>
<td>—</td>
<td>—</td>
<td>0.176380</td>
<td>0.002152</td>
<td>0.073325</td>
<td>0.585646</td>
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<tr>
<td>$\infty$</td>
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<td>0.073651</td>
<td>0.586320</td>
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</tr>
</tbody>
</table>

$\alpha_t = 0$, for all $t$

$s = 1.0$

$\rho = 0.01, \gamma = 0.01, \beta = 0.7, \delta = 0, a_t = 0.01$, $t \geq 0$

\(^9\)I am indebted to Fischer Black for suggesting this result. In a sense, Feldstein’s two effects are reversed here: this reduction in the capital stock comes about solely because of the induced retirement effect [cf. Feldstein (1977a)].
6. Summary

The discussion of the Diamond and Samuelson–Gale models in section 2 makes clear that there will normally be institutional debt or capital in an infinite-horizon, overlapping-generations model and that the nature of the model is greatly affected by the assumptions governing the growth rates of debt or capital. If one thinks of the institution as being the government, Diamond's assumption that per capita debt (or capital) is fixed seems more realistic than the assumption that the propensity to save out of interest payments on institutional debt or capital is unity. Alternatively, if the institution is a private, infinitely-lived organization, the Samuelson–Gale assumptions may be appropriate. The analysis of savings plans in section 3 gives a concrete example of the section 2 story. Section 4 shows the similarity between pay-as-you-go social security and the treatment of internal debt in Diamond (1965); pay-go reduces the capital stock in the short run and in the long run. The only exception to this comes in the Samuelson–Gale model, with the 'institution' owning capital, where the steady-state capital–labour ratio is unaffected. Section 5 demonstrates that Feldstein's theoretical analyses of social security assume a program that is infeasible, unless the economy is at the golden rule, in which case the long-run capital–labour ratio is unaltered by the introduction of the social-security program.

I have employed two-period overlapping-generations models to study some issues in public finance. Others have used similar models to explore the micro foundations of money [Wallace (1980), Balasko and Shell (1980, 1981a, 1981b)] and issues in international trade [Gale (1971, 1972), Green (1972)]. Several researchers have concentrated on the Samuelson–Gale model, and, in particular, have dealt with the Samuelson case in which the interest rate is less than the population growth rate. Recognition of the differences between the Diamond and Samuelson–Gale models may be as important in these areas as it is in public finance.

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