Economics 393  
Monopoly with price discrimination

Consider a monopoly that can sell its output in two markets, with resale between the markets impossible. Let the inverse demand functions be $P_i(Q_i), i = 1, 2$. Write revenue in each market as $R_i(Q_i) = P_i(Q_i)Q_i, i = 1, 2$. Assume the firm produces its output in one plant, with total cost equal to $C(Q_1 + Q_2)$. Profits are

$$\text{Profits}(Q_1, Q_2) = R_1(Q_1) + R_2(Q_2) - C(Q_1 + Q_2).$$

Assume Profits are strictly concave in outputs and optimal outputs are positive. From the first-order conditions we have:

$$\frac{dR_1(Q_1)}{dQ_1} = C''(Q_1 + Q_2) = \frac{dR_2(Q_2)}{dQ_2}.$$

To maximize Profits, the firm should divide output between the two markets to equalize marginal revenue in each market and marginal revenue should equal marginal cost. In either market

Marginal revenue $= P + Q \frac{dP}{dQ} = P \left(1 + \frac{Q \frac{dP}{dQ}}{P \frac{dP}{dQ}}\right) = P \left(1 - \frac{1}{\eta}\right)$

where $\eta$ is the absolute value of the price elasticity of demand: $\eta \equiv -(dQ/dP)(P/Q)$.\(^1\) Then equalizing marginal revenue in each market means

$$P_1 \left(1 - \frac{1}{\eta_1}\right) = P_2 \left(1 - \frac{1}{\eta_2}\right).$$

Thus, if the demand in market 1 is more inelastic than the demand in market 2, that is, $\eta_1 < \eta_2$, then

$$1 - \frac{1}{\eta_2} > 1 - \frac{1}{\eta_1} \quad \text{and} \quad P_1 > P_2.$$

In the theatre question in Workouts, non-students have a more inelastic demand than students, and so the theatre finds it profitable to charge non-students a higher price than students.

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\(^1\)Since a profit-maximizing monopolist will never produce at an output where marginal revenue is negative one implication of this formula is that, at the optimal monopoly output, “demand is price elastic” — $\eta > 1$. 

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