Human Capital and Capital Markets

Assume a two-period, one-person model. The person has a time endowment of $T$ in period 1 which can be allocated to schooling, $s_1$, or to work, $T - s_1$, for a real wage rate $w_1$. The person’s hours of work are fixed in period 2 but total real earnings, $e_2$, depend on schooling in the following ways:

\[
\begin{align*}
e_2(0) &= 0 \\
e_2'(s_1) &> 0 \\
e_2''(s_1) &< 0.
\end{align*}
\]

Denoting the person’s lifetime consumption plan by $(c_1, c_2)$ the person’s preferences can be represented by $U(c_1, c_2)$.

(a) Assume the person has access to a perfect capital market with real interest rate $r$. Then the person’s lifetime budget constraint is

\[
c_1 + \frac{c_2}{1 + r} = w_1 (T - s_1) + \frac{e_2(s_1)}{1 + r}.
\]

In this setting the person could optimize by picking $s_1$ to maximize the right-hand-side of the budget constraint and then, given the optimal schooling value of $s_1^*$, pick $(c_1, c_2)$ to maximize $U(c_1, c_2)$. Equivalently, the person could solve

\[
\max_{c_1, s_1} U(c_1, (1 + r)(w_1(T - s_1) - c_1) + e_2(s_1)).
\]

The first-order conditions are

\[
\begin{align*}
U_1 + U_2 (1 + r) (-1) &= 0 \\
U_2 [(1 + r) w_1 (-1) + e_2'(s_1)] &= 0.
\end{align*}
\]

These equations can be written as

\[
\frac{e_2'(s_1)}{w_1} = 1 + r = \frac{U_1}{U_2}.
\]

The second equation is the familiar MRS condition; the first says that at the optimal level of schooling the marginal return to further education should equal the real rate of interest.
(b) Now assume that capital markets are imperfect in the sense that the person cannot borrow against second-period earnings. If the person doesn’t want to borrow against second-period earnings, that is, \( w_1 (T - s_1^*) - c_1^* \geq 0 \), then this imperfection in the capital market doesn’t matter. If \( w_1 (T - s_1^*) - c_1^* < 0 \) then this capital market imperfection will matter. The separability between the schooling decision and the consumption allocation decision breaks down and the budget constraints become the following two equations.

\[
\begin{align*}
  c_1 &= w_1 (T - s_1) \\
  c_2 &= e_2 (s_1),
\end{align*}
\]

so the person’s optimization problem can be written as

\[
\max_{s_1} U (w_1 (T - s_1), e_2 (s_1)).
\]

The first-order condition is

\[
U_1 w_1 (-1) + U_2 e_2' (s_1) = 0 \text{ or } e_2' (s_1) = \frac{U_1}{U_2} > 1 + r.
\]

Here the chosen level of \( s_1 \) lies below \( s_1^* \), the chosen level of \( c_1 \) will be less than in (a) and utility will be lower than in (a). The competitive equilibrium is not Pareto efficient and some would argue this model makes a case for government subsidization of education.

**Tax policy**

Consider the choice between a proportional income tax \( t^e = tr \) and a proportional consumption tax \( t^c \) in this kind of model. With a perfect capital market as in (a) an interest tax distorts both the human capital decision and the consumption plan:

\[
\frac{e_2' (s_1)}{w_1} = 1 + (1 - tr) r = \frac{U_1}{U_2}
\]

The person will overinvest in human capital as \( s_1 \) is increased from the point where one plus the rate of return on human capital equals \( 1 + r \) to the point where this equals the lower number of \( 1 + (1 - tr) r \). John Driffill and Harvey Rosen (International Economic Review, 1983) argue that with perfect capital markets and a human capital choice an income tax is much less efficient than a consumption tax.

With imperfect capital markets that matter as in (b) we start from an inefficient equilibrium with no taxes in which people are choosing less than the socially optimal level of schooling because they are liquidity constrained when they are young. In a multi-period life cycle model with this kind of problem the person has a higher marginal utility of money when young than she has when she is older. And changes in tax policy that redistribute money from later in the life cycle to earlier in the life cycle generate a first-order welfare gain.