1. Consider a risk-averse individual with initial wealth $w_0$ and a Bernoulli utility function $u(w)$ who must decide whether and for how much to insure his car. Assume the probability that he will not have an accident is $\pi$. In the event of an accident, he incurs a loss of $L < w_0$ in damages. Suppose that insurance is available at an actuarially fair price, that is, one that yields insurance companies zero expected profits; denote the price of $1$ worth of insurance coverage by $p$. Assume it costs nothing to run an insurance company.

(i) Write down the individual’s expected utility if he purchases $x$ dollars of insurance.

(ii) Prove that in the competitive equilibrium in the insurance market $p = 1 - \pi$.

(iii) How much insurance will the individual purchase in the competitive equilibrium?

(iv) Does this individual’s demand for insurance slope downward with respect to the price of insurance at the point where insurance is priced fairly? Defend your answer carefully.

**ANSWER**

(i) The individual’s expected utility is

$$f(x, p, \pi, w_0, L) \equiv \pi u(w_0 - px) + (1 - \pi) u(w_0 - px - L + x)$$

(ii) In a competitive equilibrium the firm’s expected profit on each dollar of insurance is zero, so

$$\pi p + (1 - \pi) (p - 1) = 0 \quad \text{or} \quad p = 1 - \pi.$$ 

(iii) To find the optimal value of $x$ given the parameters of the problem set

$$f_1(x, p, \pi, w_0, L) = 0$$

and solve for $x$ at $p = 1 - \pi$. Thus
\begin{align*}
  (-p) \pi u' (w_0 - px) + (1 - \pi) (1 - p) u' (w_0 - px - L + x) &= 0 \\
  u' (w_0 - px) &= u' (w_0 - px - L + x) \quad \text{and thus} \\
  w_0 - px &= w_0 - px - L + x \quad \text{or} \\
  x &= L.
\end{align*}

(iv) We want to know

\[ \text{Sign} \left. \frac{dx}{dp} \right|_{p=1-\pi} \]

Note on comparative statics

Suppose an agent is maximizing her objective function \( f(x, a) \), where \( x \) is the choice variable and \( a \) is a parameter. Assuming the maximization problem is well-behaved, and all we are interested in is the sign of \( dx/da \) then the method of comparative statics (the implicit function theorem) tells us that

\[ \text{Sign} \left. \frac{dx}{da} \right| \quad = \quad \text{Sign} \ f_{12} (x, a). \]

Why? The first-order condition for this problem is

\[ f_1 (x, a) = 0 \]

which implicitly defines the optimal level of \( x \) as a function of \( a \). Taking a total differential of this equation obtain

\[ f_{11} (x, a) \, dx + f_{12} (x, a) \, da = 0 \quad \text{or} \\
  \left. \frac{dx}{da} \right| = -\frac{f_{12} (x, a)}{f_{11} (x, a)}. \]

The result follows immediately from this equation and that the second-order conditions for the maximization require \( f_{11} (x, a) < 0 \).

In the context of question 1 then

\[ \text{Sign} \left. \frac{dx}{dp} \right|_{p=1-\pi} = \text{Sign} \ f_{12} (x, p = 1 - \pi, \pi, w_0, L) \]

From above

\[ f_1 (x, p, \pi, w_0, L) = (-p) \pi u' (w_0 - px) + (1 - \pi) (1 - p) u' (w_0 - px - L + x), \]
Checking second-order conditions

\[ f_{11}(x, p, \pi, w_0, L) = (-p)^2 \pi u''(w_0 - px) + (1 - \pi)(1 - p)^2 u''(w_0 - px - L + x) \]

which must be negative if \( u(w) \) is strictly concave, that is, \( u''(w) < 0 \).

Then, applying the method of comparative statics,

\[ f_{12}(x, p, \pi, w_0, L) = -\pi u'(w_0 - px) + (-p) \pi u''(w_0 - px)(-x) \]
\[ - (1 - \pi) u'(w_0 - px - L + x) + (1 - \pi)(1 - p) u''(w_0 - px - L + x)(-x) \]

But at \( p = 1 - \pi \) we have proved \( x = L \) so

\[ f_{12}(x, p = 1 - \pi, \pi, w_0, L) = -u'(w_0 - (1 - \pi)L) < 0 \]

Thus, at the competitive equilibrium, an increase in the price of insurance lowers the demand for insurance. Recall from Econ 290 that ordinary demand functions do not slope downwards without some restrictions. That we can prove that the demand for insurance is downward sloping here is another example of the power of the independence axiom in the expected utility model.

2. Recast the model in question 1 as a two-good Econ 290 model. Denote wealth in the ‘good’ state (no accident) by \( w_g \) and wealth in the ‘bad’ state (an accident occurs) by \( w_b \).

(i) In a diagram with \( w_g \) on the horizontal axis and \( w_b \) on the vertical axis, draw the individual’s budget constraint with \( p \) equal to its competitive equilibrium value. What is the absolute value of the slope of this budget constraint?

(ii) Write down the individual’s expected utility as a function of \( w_g \) and \( w_b \). Calculate the MRS between \( w_g \) and \( w_b \).

(iii) Use (i) and (ii) here in question 2 to verify your answer to question 1 (iii) above.

**ANSWER**

(i) If the individual chooses to buy no insurance then \( w_g = w_0 \) and \( w_b = w_0 - L \), so this point anchors the budget constraint. Buying negative insurance is not allowed so the budget constraint slopes up to the left of \( w_0, w_0 - L \). The price of 1 dollar of insurance is \( p \). This means that if the individual buys 1 dollar of insurance then she gives up \( p \) dollars in either state and gets back 1 in the bad state. The absolute value of the slope of the slope of the budget constraint is \( (1 - p)/p \).
(ii) The individual’s expected utility, EU, is

\[ \text{EU} = \pi u(w_g) + (1 - \pi)u(w_b). \]

The MRS between \( w_g \) and \( w_b \) is

\[ \text{MRS}_{w_gw_b} = \frac{\pi u'(w_g)}{(1 - \pi)u'(w_g)}. \]

(iii) So in a competitive equilibrium \( p = 1 - \pi \), the slope of the budget constraint is \((1-p)/p = \pi/(1-\pi)\), and thus

\[ \frac{\pi u'(w_g)}{(1 - \pi)u'(w_g)} = \frac{\pi}{1 - \pi} \text{ or } u'(w_g) = u'(w_b). \]

But, as in question 1, we are assuming \( u''(w) < 0 \), so \( u'(w_g) = u'(w_b) \) implies \( w_g = w_b \) which implies

\[ w_0 - px = w_0 - px - L + x \text{ or } x = L. \]

This is the same as the answer in question 1 (iii).