This week you studied the note on the web site for Pareto efficient provision of public goods and then did problems 36.1 to 36.6 in Workouts. You have seen examples of the Samuelson rule for efficient provision of public goods. Analytically, all of the examples use quasilinear or homothetic preferences or both, which is why we went through these preferences in Mas-Colell, Whinston and Green (MWG). Occasionally, majority voting with a head tax can deliver the efficient level of public goods — the rink question in 36.1 is one example. The restaurant problem — 36.2 — has nothing to do with public goods but it does highlight a problem with the ‘splitting the bill’ mechanism in restaurants, even if everyone has the same income and the same preferences. Many of the best examples of public good problems occur in households — see 36.4 and 36.6. 36.6 makes the important point that moving along the household’s utility possibility frontier in the direction of increasing the utility of the person with the weaker preference for public goods will lead to less public good. 36.5 shows that public goods issues may arise in production.

Recall that the Samuelson kind of public good is a good where consumption is non-rival (one person’s use of the good doesn’t reduce the amount available to others) and the good is non-excludable (once a quantity of the good is provided, that quantity is available to everyone). Some public goods can be provided by a group of private agents — e.g., an access road to a group of cottages around a lake, or public goods within a household. In the opposite direction, sometimes governments provide purely private goods. Some commentators have argued that education is (primarily) a private good that is, at some levels, provided by the government. Problem 36.7 deals with this kind of example. Aerobics lessons are a purely private good that the government has decided to provide. The issues are, given majority voting, what are the consequences of different tax systems for economic efficiency and income redistribution. Problem 36.7 works with the Cobb-Douglas utility function $A^{1/2}B^{1/2}$. Numerically, it is easier to work with the square of this function; recall that strictly increasing transformations of utility functions don’t affect ordinary demand functions. The example below switches problem 36.7 so there is a majority of rich people, rather than a majority of poor people. This highlights some of the problems with public provision of private goods in a democracy.
Public provision of private goods

Suppose the citizens of the City of Toronto care about only two things — aerobics lessons (A) and bread (B). Everyone has the same utility function

\[ U = AB. \]

The price of an aerobics lesson is $2 and the price of a loaf of bread is $1. Two million of Toronto’s citizens are rich and have an income of $100 each; one million are poor and have an income of $50 each. (a) Find the utility level of each rich person and each poor person in a private equilibrium with no government intervention. (b) Now suppose the government decides to provide aerobics lessons publicly. Assume that majority vote determines the number of aerobics lessons, which must be the same for every citizen. Further assume that the government must balance its budget — what it collects in taxes must cover the total costs of providing aerobics lessons to all citizens. Find the utility level of each poor person and each rich person if the government levies a “head” tax — that is, a tax that is the same number of dollars per person, whether rich or poor. (c) Find the utility level of each poor person and each rich person if everything is as in (b) except the government uses a proportional income tax.

**ANSWER**

(a) The utility function for this question is in the Cobb-Douglas family, \( u(A, B) = A^a B^b \) and we know

\[
A(p_A, p_B, \text{wealth}) = \frac{a}{a + b} \frac{\text{wealth}}{p_A} = \frac{1}{2} \text{wealth},
\]

\[
B(p_A, p_B, \text{wealth}) = \frac{b}{a + b} \frac{\text{wealth}}{p_B} = \frac{1}{2} \text{wealth}.
\]

The private-equilibrium row in the table below follows directly from these equations.

(b) For the government in this question revenue must equal expenditure. With a head tax this means

\[
(3 \text{ million})(\text{head tax}) = (3 \text{ million})p_A A \text{ or, the head tax } = 2A.
\]

Since the rich have a majority their budget constraint is the one that matters.

\[
B = \text{Income} - \text{head tax} = 100 - 2A \text{ or } 2A + B = 100.
\]

With this budget constraint the rich choose \( A = 25 \), with a head tax of 50, and \( B = 50 \). This means the poor have to live with \( A = 25 \) and no other income for bread after they pay the head tax — \( B = \text{ utility} = 0 \). These results are summarized in the head-tax row of the table.

(c) With a proportional income tax, \( t \), we have
\[ t[(2 \text{ million})(\text{income of rich}) + (1 \text{ million})(\text{income of poor})] = (3 \text{ million})2A, \]

or

\[ t = \frac{6A}{250} = \frac{3A}{125}. \]

So then the budget constraint of the rich is

\[ B = (1 - t) \text{ income} = 100 \left(1 - \frac{3A}{125}\right) = \frac{4}{5} (125 - 3A) \]

or

\[ 12A + 5B = 500. \]

Given this budget constrain the rich vote for

\[ A = \frac{1500}{12} = \frac{125}{6} \quad \text{and} \quad t = \frac{3A}{125} = \frac{1}{2}. \]

From here the results in the last row of the table follow directly.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Poor A</th>
<th>B</th>
<th>Utility</th>
<th>Rich A</th>
<th>B</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private eq.</td>
<td>25/2</td>
<td>25</td>
<td>(1/2)25^2</td>
<td>25</td>
<td>50</td>
<td>(2)25^2</td>
</tr>
<tr>
<td>Head tax</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>50</td>
<td>(2)25^2</td>
</tr>
<tr>
<td>Income tax</td>
<td>125/6</td>
<td>25</td>
<td>(5/6)25^2</td>
<td>125/6</td>
<td>50</td>
<td>(5/3)25^2</td>
</tr>
</tbody>
</table>

Comments

The private equilibrium in this example is Pareto efficient. The government is not acting to correct an inefficiency — the government’s main objective is to reduce the utility gap between rich and poor. With this in mind, the head tax row shows that sometimes governments with good intentions get into real trouble — the utility of the rich is unchanged from the private equilibrium but the utility of the poor drops to zero. The private eq. and income tax rows illustrate the classic trade-off between equity and efficiency that is a major theme in public economics courses. The income tax equilibrium delivers higher utility for the poor at the cost of lower economic efficiency and lower utility for the rich.