For questions 1, 2 and 3 you did Workouts questions 25.9, 25.10 and 25.11, and you have the answers for these three questions. Here is a more detailed answer for 25.11.


**ANSWER**

(a) Maximizing $C x(y - x)$ with respect to $x$ we obtain $x = y/2$. Maximizing $C(1 - y)(y - x)$ with respect to $y$ we obtain $y = (1 + x)/2$.

(b) Solving the two equations in (a) simultaneously we obtain $x = 1/3$ and $y = 2/3$.

(c) The key is to see the similarity between this problem and question 1 on this assignment. Below you can find a picture of voter contributions as a function of political preference, given candidate locations. Consider contributions for the candidate located at $x$. Each voter with political views at or to the left of $x$ (or at or to the right of $y$) contributes $C(y - x)$. Voters between $x$ and $y$ — “moderates” — “… contribute in proportion to the difference between their own ideological distance from their less-preferred candidate and their ideological distance from their more-preferred candidate.” Consider a voter located at $a$ where $x \leq a \leq (x + y)/2$. This voter contributes $C((y - a) - (a - x))$ which equals $C(x + y - 2a)$ to $x$'s campaign. So this voter contributes nothing if $a = (x + y)/2$ and $C(y - x)$ if $a = x$. Thus voter contributions for the candidate at $x$ fall linearly from $C(y - x)$ at location $x$ to zero at $(x + y)/2$. Thus total contributions for the candidate at $x$ equal

$$C(y - x)x + \frac{1}{4}C(y - x)^2.$$ 

Choosing $x$ to maximize this see that $x = y/3$. Total contributions for the candidate at $y$ equal

$$C(y - x)(1 - y) + \frac{1}{4}C(y - x)^2.$$ 

Choosing $y$ to maximize this see that $y = (x + 2)/3$. Solving these two equations in $x$ and $y$ we obtain $x = 1/4$ and $y = 3/4$. Note that the total contributions are $3C/16$ for each candidate.
4. This is an extension of Workouts problem 25.9. The “town” comprises people uniformly distributed along a line 30 miles long, with 100 people per mile. Travel costs are 1 dollar per person per mile. There is one bowling alley located at mile 10 and another located at mile 20. All customers are willing to pay up to 15 dollars for a night of bowling. The marginal cost of caring for a customer while at the bowling alley is 3 dollars. Ignore the fixed costs of the bowling alley — set them equal to zero. There is price competition between the bowling alleys in this question. Be careful with the demand function for each bowling alley in part (a) and think about how price competition will determine the equilibrium in (b).

(a) Assume customers pay their own transport costs to the bowling alley and each bowling alley acts to maximize its profits. Calculate total consumers’ surplus and the total profits of the two bowling alleys.

(b) Now assume the bowling alleys pay transport costs for their customers, and again, each bowling alley acts to maximize its profits. Calculate total consumers’ surplus and the total profits of the two bowling alleys.

(c) Is the setting in either (a) or (b) Pareto efficient? Defend your answer.

**ANSWER**

(a) Think about the bowling alley at mile 10. Reducing price from 15 dollars to 10 dollars will increase the number of customers by 200 for each 1 dollar reduction in price. But once price gets to 10 dollars the customers will be coming from mile 5 to mile 15. If it were to cut price to 9 it wouldn’t get the customers between mile 15 and mile 16 — they are going to be the customers of the mile-20 bowling alley, because it’s closer for them, and in the spatial equilibrium the two bowling alleys are going to be charging the same price. So if the mile-10 alley cuts price from 10 to 9 the only only new customers it attracts are the 100 customers between mile 4 and mile 5. Thus the mile-10 bowling alley has a market demand that can be written as:

\[
Q(p) = \begin{cases} 
200(15 - p) & \text{if } 10 \leq p \leq 15 \\
100(10 - p) + 1000 & \text{if } 0 \leq p < 10
\end{cases}
\]

Notice this is a continuous function but there is a kink at \( p = 10 \). There is no price discrimination in this section of the question so inverting demand we obtain average revenue as a function of \( Q \).

\[
AR(Q) = \begin{cases} 
15 - Q/200 & \text{if } 0 \leq Q \leq 1000 \\
20 - Q/100 & \text{if } 1000 < Q \leq 2000
\end{cases}
\]

With a linear average revenue, marginal revenue, MR, will also be linear and have the same price-axis intercept and twice the slope.

\[
MR(Q) = \begin{cases} 
15 - Q/100 & \text{if } 0 \leq Q \leq 1000 \\
20 - Q/50 & \text{if } 1000 < Q \leq 2000
\end{cases}
\]

Notice the kink in the demand and the average revenue functions generates a discontinuity in marginal revenue. MR falls from 5 at \( Q = 1000 \) to negative numbers for larger values of \( Q \). Since marginal cost is 3 dollars per customer there is no \( Q \) where MR equals MC.
The profit-maximizing $Q$ is 1000. Equilibrium price is 10 dollars for night of bowling. Profit per customer is $10 - 3 = 7$ dollars — so total profits for the two alleys is 14000 dollars. Consumer’s surplus is zero at miles 5, 15 and 25. Consumer’s surplus peaks at 5 dollars at each bowling alley — mile 10 and mile 20. Given that customers are uniformly distributed along the line at the rate of 100 per mile total consumers’ surplus is 5000 dollars.

(b) If the bowling alleys are paying the transport costs then the bowling alleys know where people live. (Like the casinos that send out buses to pick up people who want to come to the casino to gamble.) If the alleys could charge everyone 15 dollars for a night of bowling they would. But they can’t. Think of the person at mile 15, halfway between the bowling alleys. The cost to either alley of looking after her are 8 dollars — 5 dollars in transport costs plus 3 dollars at the alley. If the mile-10 alley charged her 9 dollars the mile-20 alley would undercut that price — so spatial price competition will make the price to her equal to 8 dollars. At mile 14 the mile-10 alley can charge 9 dollars because that is what it would cost the mile-20 alley to look after that person. So starting at mile 15 and moving towards mile 0 price rises at a dollar per mile, from at 8 dollars at mile 15. Price reaches its highest possible level of 15 dollars at mile 8 and stays at that level all the way to mile 0. Pricing for miles 15 to 30 is symmetric with pricing for miles 15 to 0. See the top graph in Figure 2.

At mile 0, profit is $15 - 10(\text{transport costs}) - 3(\text{cost at the alley}) = 2$ dollars. It’s constant at 10 dollars between miles 8 and 10 and then falls to zero at mile 15. See the middle graph in Figure 2. Remembering that there are 100 per mile total profits for the two alleys is 18600 dollars.

Since the alleys are paying transport costs here, at each location, consumer’s surplus is 15 minus price. See the last graph in Figure 2. Total consumers’ surplus is 4900 dollars.

(c) One way to go about checking if some private equilibrium is Pareto efficient is to ask what a central planner would do — figure out how much social surplus would be generated — and then compare that number with the total of consumers’ surplus and profits in the private equilibrium. Would a central planner have the person located at mile 0 bowl? Well, it costs 10 in transport costs to get this person to the mile-10 alley and then 3 at the alley, for a total of 13 dollars in costs. The person’s utility from a night of bowling is 15 dollars, so, yes, the central planner would have the person located at mile 0 bowling. And if it’s efficient to have this person bowling, it’s efficient that everyone bowls. The “town” is 30 miles long; at 100 people per mile there are 3000 people bowling, each of whom is willing to pay 15 dollars. Thus total benefits are 45000 dollars. For the 1000 people between mile 0 and mile 10, average transport costs are 5 per person — for a total of 5000 dollars. For the 500 people between mile 10 and mile 15 average transport costs are 5/2 dollars — so total costs for this group equal 1250. Given the symmetry of the town around mile 15 total transport costs for the town equal $2(5000 + 1250) = 12500$. With a 3 dollar cost per person at the alley, total alley costs are 9000 dollars. Thus net benefits = $45000 - 12500 - 9000 = 23500$ dollars. This number is much greater than the total of consumers’ surplus and profits in (a) but it equals the total of profits and consumers’ surplus in (b). So (b) is Pareto efficient; (a) is not.
Problem 25.14

Political Contribution

\[ C(y|x) \]

\[ x \]

\[ \frac{x+y}{2} \]

\[ y \]

\[ \frac{y-x}{2} \]

Voter Preference