Economics 393
Assignment 2 answers

1. In this simple setting there is no difference between the As having a *monopoly selling good 2* or *monopsony buying good 1*. The answers to (c) and (d) do not change.

2. The answers for this question are in the *Workouts* pdf with answers, which I emailed to you. Below you will see a longer version of the answers, together with the answers for the case where there is *no price discrimination* and the capacity constraint of 150 seats is imposed. There are at least two important conclusions to draw from this question. First, a monopolist that has the power to practice price discrimination could always choose not to do so. This means that the monopolist’s profits must increase looking across the same situation, with and without, price discrimination. Check this out in the table below. Second, look at the first and third rows of the table below for the “no constraint” case. The total of profits and consumers’ surplus falls with price discrimination. Ditto for rows 2 and 4. So when agents can practice price discrimination social efficiency does not always increase — it may decrease.

Student demand is

\[ Q_S = 220 - 40P_S, \]

and non-student demand is

\[ Q_N = 140 - 20P_N. \]

Variable costs are zero and fixed costs are 500 dollars per night.

*Monopoly: no price discrimination*: here the theatre must operate with the constraint that \( P = P_S = P_N \). Adding up the two demands we have \( Q = 360 - 60P \) or average revenue equals price equals \( 6 - Q/60 \). Then

\[
\text{Profits}(Q) = \left( 6 - \frac{Q}{60} \right) Q - 500
\]

\[
\frac{d\text{Profits}}{dQ} = 6 - \frac{Q}{30}
\]

\[
\frac{d^2\text{Profits}}{dQ^2} = -\frac{1}{30} < 0.
\]
Profits are strictly concave in output — setting the slope of Profits with respect to output equal to zero we see $Q = 180$. So then price equals $6 - \frac{Q}{60} = 3$ and Profits are $(3)(180) - 500 = 40$. From the demand lines $Q_S = 100$ and $Q_N = 80$. If the theatre had only 150 seats the seating capacity constraint would bind: $Q = 150; \ P = 6 - \frac{150}{60} = 7/2; \ $Profits $= (7/2)(150) - 500 = 25$. And from the demand lines $Q_S = 80$ and $Q_N = 70$. Note: adding the binding capacity constraint must lower profits.

Monopoly: with (some) price discrimination: here $P_S$ can be different from $P_N$.

\[
\begin{align*}
P_S &= \frac{11}{2} - \frac{Q_S}{40} \\
P_N &= 7 - \frac{Q_N}{20}
\end{align*}
\]

Profits $= P_S Q_S + P_N Q_N - 500 = \left(\frac{11}{2} - \frac{Q_S}{40}\right) Q_S + \left(7 - \frac{Q_N}{20}\right) Q_N - 500$

\[
\begin{align*}
\frac{\partial \text{Profits}(Q_S, Q_N)}{\partial Q_S} &= \frac{11}{2} - \frac{Q_S}{20} \\
\frac{\partial \text{Profits}(Q_S, Q_N)}{\partial Q_N} &= 7 - \frac{Q_N}{10},
\end{align*}
\]

and the Hessian of Profits($Q_S, Q_N$) is

\[
\begin{pmatrix}
-1/20 & 0 \\
0 & -1/10
\end{pmatrix},
\]

which is a negative definite matrix. Thus Profits are strictly concave in sales to students and non-students. Setting the partial derivatives equal to zero we obtain $Q_S = 110$ and $Q_N = 70$. Then the corresponding prices are $P_S = \frac{11}{2} - 110/40 = 11/4$ and $P_N = 7 - 70/20 = 7/2$. Profits are $95/2$.

If the theatre can practice price discrimination but it has a capacity constraint of 150 seats then we can write $Q_N = 150 - Q_S$. Here

\[
\begin{align*}
\text{Profits}(Q_S) &= \left(\frac{11}{2} - \frac{Q_S}{40}\right) Q_S + \left(7 - \frac{150 - Q_S}{20}\right) (150 - Q_S) - 500 \\
\frac{d \text{Profits}}{d Q_S} &= \frac{11}{2} - \frac{Q_S}{40} - \frac{Q_S}{40} + \frac{150 - Q_S}{20} - \left(7 - \frac{150 - Q_S}{20}\right)
\end{align*}
\]

It is easy to check that Profits are strictly concave in $Q_S$. Setting the slope of Profits with respect to $Q_S$ equal to zero we find $Q_S = 90$. Therefore, $Q_N = 150 - 90 = 60$. Prices are $P_S = 13/4, \ P_N = 4$ and Profits equal $65/2$.

The results are summarized in the table below.
3. (a) We can solve this problem by writing Profits as a function of output $Q$ and then finding the output level that maximizes profits. There is no price discrimination in part (a) so

$$\text{Average revenue}(Q) = p = \frac{220000}{4000} - \frac{Q}{4000} = 55 - \frac{Q}{4000}$$

Thus

$$\text{Profits}(Q) = \left(55 - \frac{Q}{4000}\right)Q - 5Q - 1500000 = 50Q - \frac{Q^2}{4000} - 1500000$$

This is a strictly concave function of $Q$. Setting the derivative of this with respect to $Q$ equal to zero we see that Profits are maximized at $Q = 100000$, and then from the price equation above $p = 30$ dollars per case and Profits are 1000000.

(b) Here the trade restrictions between US and Canada allow the firm to charge different prices either side of the border. If $Q$ cases are sold in Ontario Molson can sell its plant’s capacity, 200000, minus $Q$ cases in the US. Its Profits are

$$\text{Profits}(Q) = \left(55 - \frac{Q}{4000}\right)Q + 25(200000 - Q) - (5)(200000) - 1500000 = 30Q - \frac{Q^2}{4000} + 2500000$$

Setting the derivative of this with respect to $Q$ equal to zero we see that Profits are maximized at $Q = 60000$ cases sold in Ontario, and then from the price equation above $p = 40$ dollars per case in Ontario and Profits in Ontario and the US combined are 3400000 Canadian dollars.