Economics 393  
Assignment 11 answer

Read the note on production functions on the web site and then work through the first 12 pages of the Wright survey (up to the beginning of section 2.2). Try to draw Figure 1 for the case where the matching technology is Cobb Douglas e.g. \( \mu(n_1, n_2) = n_1^\beta n_2^{1-\beta}, 0 < \beta < 1 \). Using the note on production functions this means that the \( \alpha(n) \) function in the Wright survey is

\[
\alpha(n) = n^\beta \text{ where } 0 < \beta < 1.
\]

**ANSWER**

From Wright et al. we know

\[
p = \epsilon c + (1 - \epsilon)u \text{ where } \epsilon \equiv \frac{n \alpha'(n)}{\alpha(n)},
\]

and \( \alpha(n) \) is the probability that a seller finds a buyer. In this question

\[
\frac{n \alpha'(n)}{\alpha(n)} = \frac{n \beta n^{\beta-1}}{n^\beta} = \beta,
\]

which is a constant, independent of \( n \), the ratio of buyers to sellers. Thus the tangencies between buyer and seller indifference curves in an \( n-p \) diagram form a horizontal line higher than \( c \) and lower than \( u \).

The harder part of this question is to determine the shapes of buyer and seller indifference curves. Start with the seller. For any given \( p \) seller expected profit is higher the higher the ratio of buyers to sellers, which is \( n \) — moving to the right in Figure 1 increases seller expected profit. The equation for seller indifference curves or seller iso-profit lines in \( n-p \) space is

\[
V_s = \alpha(n)(p - c) \text{ or } p = \frac{V_s}{\alpha(n)} + c
\]

So here
\[ p = \frac{V_s}{\alpha(n)} + c = n^{-\beta}V_s + c \]

\[ \frac{dp}{dn} = -\beta n^{-1-\beta}V_s < 0 \]

\[ \frac{d^2p}{dn^2} = -\beta(\beta - 1)n^{-\beta - 2}V_s = \beta(1 + \beta)n^{-\beta - 2}V_s > 0 \]

These results tell us that seller iso-profit lines are downward-sloping and “better sets” for sellers are strictly convex.

Now turn to buyers. The equation for buyer indifference curves in \( n-p \) space is

\[ V_b = \frac{\alpha(n)}{n} (u - p) \] or \( p = -\frac{n}{\alpha(n)} V_b + u \)

So here

\[ p = -\frac{n}{\alpha(n)} V_b + u = -n^{1-\beta}V_b + u \]

\[ \frac{dp}{dn} = -(1 - \beta)n^{-\beta}V_b < 0 \]

\[ \frac{d^2p}{dn^2} = \beta(1 - \beta)n^{-\beta - 1}V_b > 0 \]

These results tell us that buyer indifference curves are downward-sloping and, since buyer expected utility increases moving to the left in Figure 1, “better sets” for buyers are not strictly convex. This raises the possibility that tangencies between buyer indifference curves and sellers iso-profit curves may not be optimal points. For tangencies to be optimal we need that seller iso-profit curves are more convex than buyer indifference curves, that is, \( d^2p/dn^2 \) in (1) exceeds \( d^2p/dn^2 \) in (2).

Since equilibrium price \( p = \epsilon c + (1 - \epsilon)u \), in equilibrium

\[ V_s = \alpha(n)(p - c) = \alpha(n)(\epsilon c + (1 - \epsilon)u - c) = \alpha(n)(1 - \epsilon)(u - c) = (1 - \beta)n^\beta(u - c) \]

and

\[ V_b = \frac{\alpha(n)}{n} (u - p) = \frac{\alpha(n)}{n} (u - \epsilon c - (1 - \epsilon)u) = \frac{\alpha(n)}{n} \epsilon(u - c) = \beta n^{\beta - 1}(u - c). \]

Substitute the expression for \( V_s \) into (1) to obtain the rate of change of the slope of the seller’s iso-profit curve at an equilibrium point.

\[ \frac{d^2p}{dn^2} = \beta(1 + \beta)n^{-\beta - 2}V_s = \beta(1 + \beta)n^{-\beta - 2}(1 - \beta)n^\beta(u - c) \]

\[ = \beta(1 + \beta)(1 - \beta)n^{-2}(u - c). \]
Substitute the expression for $V_b$ into (2) to obtain the rate of change of the slope of the buyer’s indifference curve at an equilibrium point.

\[
\frac{d^2 p}{dn^2} = \beta (1 - \beta) n^{-\beta - 1} V_b = \beta (1 - \beta) n^{-\beta - 1} \beta n^{\beta - 1} (u - c) = \beta^2 (1 - \beta) n^{-2} (u - c).
\]

(4)

And then

\[
\frac{d^2 p}{dn^2} \text{ on } V_s - \frac{d^2 p}{dn^2} \text{ on } V_b = \beta (1 + \beta) (1 - \beta) n^{-2} (u - c) - \beta^2 (1 - \beta) n^{-2} (u - c) = \beta (1 - \beta) n^{-2} (u - c) > 0.
\]

So seller iso-profit curves are more convex than buyer indifference curves, the equilibria are Pareto efficient and Figure 1 looks like the picture on the next page.
Figure 1 in Wright et al. with Cobb-Douglas alpha(n)

Red curve is iso-profit

Blue curve is buyer indifference curve

Green line is set of tangencies