Econ 393
Test 2

Instructions: You need paper (lined if possible) and a pen or a pencil to write this test. You may answer the questions in any order you like. You should start each question on a new page. You must write your answers; typed answers will not be accepted. When you are finished answering the questions, please order the pages so your answers to question 1 are first, and then your answers to question 2, etc. Then, in a single email message, send an image of each page to me at jburbidg@uwaterloo.ca. Please put Econ 393, your name and your id number in the subject line of your email. The deadline for submitting your answers is 6:00 pm Tuesday July 7th, Toronto time. The marks allocated to each question are shown in brackets.

1. Suppose the citizens of the City of Toronto care about only two things — aerobics lessons (A) and bread (B). Everyone has the same utility function

\[ U = AB. \]

The price of an aerobics lesson is $2 and the price of a loaf of bread is $1. Two million of Toronto’s citizens are poor and have an income of $50 each; one million are rich and have an income of $100 each. Fill out a table like the one below for each of the following cases.

(i) (2 marks) The private equilibrium with no government intervention.

(ii) (2 marks) The government decides to provide aerobics lessons publicly. Assume that majority vote determines the number of aerobics lessons, which must be the same for every citizen. Further assume that the government must balance its budget with a head tax.

(iii) (3 marks) Everything is as it was in (ii) except the government uses a proportional income tax.

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2. Suppose the Government of Ontario has declared the roof of the old rink in New Liskeard to be unsafe and has condemned the building. The town has decided to hold a referendum on building a new rink. If the referendum passes everyone’s income will be taxed at a proportional rate $0 < t < 1$ to cover the costs of a new rink. If the rink is of size $G$ square feet, the total costs are $p_G G$. Everyone of the $N$ citizens of New Liskeard has a wealth of $w$. Apart from the rink there is a private good, $X$, which has a price of one dollar per unit of $X$. Everyone’s utility function is

$$U(X, G) = X + f(G), f'(G) > 0, f''(G) < 0.$$ 

(i) (2 marks) What is the Pareto efficient level of $G$?
(ii) (3 marks) Will the referendum lead to the Pareto efficient level of $G$? Defend your answer carefully.

3. A factory pumps waste into a lake. If $X$ is the amount of waste, the factory’s profits are

$$20X - X^2.$$ 

The lake may also used for recreational swimming by 2 people, each of whom has the utility function

$$u(C, Y, X) = C + 8Y - Y^2 - XY,$$

where $C$ is dollars spent on consumption goods, $Y$ is hours per day spent swimming in the lake and, as above, $X$ is the amount of waste the factory pumps into the lake. Assume each person has 40 dollars to spend per day and swimming in the lake is free.

(i) (2 marks) Calculate factory profits ($\pi$), the utility of each person ($u$), and social utility $Z \equiv \pi + 2u$ in the private equilibrium.
(ii) (2 marks) What is the socially efficient level of $X$? Justify your answer.

4. For question 1 on assignment 8, calculate the competitive equilibrium values of the following derivatives:

(i) (2 marks) $dx/dp$;
(ii) (2 marks) $dx/dw_0$.

5. (3 marks) Briefly explain the role the independence axiom plays in the expected utility theorem.