Instructions: You need paper (lined if possible), a ruler, and a pen or a pencil to write this quiz. You may answer the questions in any order you like. You should start each question on a new page. You must write your answers; typed answers will not be accepted. When you are finished answering the questions, please order the pages so your answer to question 1 is first, and then your answer to question 2. Then, in a single email message, send an image of each page to me at jburbidg@uwaterloo.ca. Please put Econ 393, your name and your id number in the subject line of your email. The deadline for submitting your answers is 6:00 pm Tuesday July 28th, Toronto time. The marks allocated to each question are shown in brackets.

1. (2 marks for each part) Wright et al. assume the matching technology in their model is like the standard neoclassical production function — increasing, concave and with constant returns to scale. \( \mu(n_b, n_s) \) is the number of meetings that occur given \( n_b \) buyers and \( n_s \) sellers. They denote the probability that a seller meets a buyer by \( \alpha(n) \), where \( n = n_b/n_s \).

(i) Derive the \( \alpha \) function from the \( \mu \) function.
(ii) Prove that the probability that a buyer meets a seller is \( \alpha(n)/n \).
(iii) Wright assumes that \( \alpha(0) = 0, \alpha'(n) > 0, \alpha''(n) < 0 \) and \( \lim_{n \to 0} \alpha'(n) = \infty \). As precisely as you can draw \( \alpha(n) \) as a function of \( n \).
(iv) Given the assumptions above, prove \( \alpha(n)/n \) is decreasing in \( n \).

**ANSWER**

(i)

For all \( t > 0 \), \( \mu(tn_b, tn_s) = t\mu(n_b, n_s) \) CRTS assumed

Set \( t = 1/n_s, \mu \left( \frac{n_b}{n_s}, 1 \right) = \frac{\mu(n_b, n_s)}{n_s} \)

= the prob. a seller finds a buyer = \( \alpha(n) \)

(ii) Using (i) we have

The prob. a buyer finds a seller = \( \frac{\mu(n_b, n_s)}{n_b} = \frac{n_s\mu(n_b/n_s, 1)}{n_b} = \frac{\mu(n_b/n_s, 1)}{n_b/n_s} = \alpha(n)/n. \)
From the picture above, at any point \( n_0 > 0 \), \( \alpha'(n_0) < \alpha(n_0)/n_0 \). Thus \( \alpha(n)/n \) is decreasing in \( n \).

2. (2 marks for each part) Wright discusses the “market utility approach” in which risk-neutral firms maximize expected profits, \( V_s \), subject to a minimum level of expected utility for each buyer, \( V_b \).

(i) Write \( V_s \) and then \( V_b \) in terms of \( \alpha(n), p, c, u \) and \( n \).

(ii) Prove that in equilibrium

\[
p = \varepsilon c + (1 - \varepsilon) u,
\]

where \( \varepsilon = \alpha'(n)/\alpha(n) \).

(iii) How are \( p \) and \( n \) determined in equilibrium?

(iv) Why is this equilibrium Pareto efficient?

**ANSWER**

(i) 

\[
V_s = \alpha(n)(p - c) \\
V_b = \frac{\alpha(n)}{n}(u - p)
\]
(ii) One could use Lagrange’s Method to maximize expected profits, $V_s$, subject to a minimum level of expected utility for each buyer, $V_b$. But it’s easier to solve the $V_b$ equation above for $p$,

$$p = u - \frac{nV_b}{\alpha(n)}$$

to rewrite $V_s$ as a function of $n$.

$$V_s(n) = \alpha(n)(u - c) - nV_b$$

Setting $V_s'(n) = 0$

$$\alpha'(n)(u - c) = V_b = \frac{\alpha(n)}{n}(u - p)$$

$$\varepsilon(u - c) = u - p$$

$$p = \varepsilon c + (1 - \varepsilon)u.$$  

(iii) Since $\varepsilon$ is a function of $n$ the price equation above is one equation in $p$ and $n$. We need another equation to complete the model. Wright et al. suggest at least two ways. First, all the possible $N_b$ buyers and all the possible $N_s$ sellers might participate in the search equilibrium, in which case $n = N_b/N_s$ is the second equation that completes the model. Wright says another way is to add a participation cost for buyers or sellers to the model. For example, if it costs sellers $k$ to participate in the market, free entry of sellers will make

$$k = V_s = \alpha(n)(p - c).$$

(iv) The model assumes sellers maximize expected profits, $V_s$, subject to a minimum level of expected utility for each buyer, $V_b$. By its structure, the model must generate a Pareto efficient allocation — the firms are doing the same thing a central planner would do to achieve a Pareto efficient allocation.