Econ 393
Quiz 9

Instructions: You need paper (lined if possible), a ruler, and a pen or a pencil to write this quiz. You may answer the questions in any order you like. You should start each question on a new page. You must write your answers; typed answers will not be accepted. When you are finished answering the questions, please order the pages so your answer to question 1 is first, and then your answer to question 2. Then, in a single email message, send an image of each page to me at jburbridg@uwaterloo.ca. Please put Econ 393, your name and your id number in the subject line of your email. The deadline for submitting your answers is 6:00 pm Tuesday July 28th, Toronto time. The marks allocated to each question are shown in brackets.

1. (2 marks for each part) Wright et al. assume the matching technology in their model is like the standard neoclassical production function — increasing, concave and with constant returns to scale. \( \mu(n_b, n_s) \) is the number of meetings that occur given \( n_b \) buyers and \( n_s \) sellers. They denote the probability that a seller meets a buyer by \( \alpha(n) \), where \( n = n_b/n_s \).
   (i) Derive the \( \alpha \) function from the \( \mu \) function.
   (ii) Prove that the probability that a buyer meets a seller is \( \alpha(n)/n \).
   (iii) Wright assumes that \( \alpha(0) = 0, \alpha'(n) > 0, \alpha''(n) < 0 \) and \( \lim_{n \to 0} \alpha'(n) = \infty \). As precisely as you can draw \( \alpha(n) \) as a function of \( n \).
   (iv) Given the assumptions above, prove \( \alpha(n)/n \) is deceasing in \( n \).

2. (2 marks for each part) Wright discusses the “market utility approach” in which risk-neutral firms maximize expected profits, \( V_s \), subject to a minimum level of expected utility for each buyer, \( V_b \).
   (i) Write \( V_s \) and then \( V_b \) in terms of \( \alpha(n), p, c, u \) and \( n \).
   (ii) Prove that in equilibrium
   \[
   p = \varepsilon c + (1 - \varepsilon)u,
   \]
   where \( \varepsilon = \alpha'(n)n/\alpha(n) \).
   (iii) How are \( p \) and \( n \) determined in equilibrium?
   (iv) Why is this equilibrium Pareto efficient?