Instructions: You need paper (lined if possible), a ruler, and a pen or a pencil to write this quiz. You may answer the questions in any order you like. You should start each question on a new page. You must write your answers; typed answers will not be accepted. When you are finished answering the questions, please order the pages so your answer to question 1 is first, then your answer to question 2, etc. Then, in a single email message, send an image of each page to me at jburbidg@uwaterloo.ca. Please put Econ 393, your name and your id number in the subject line of your email. The deadline for submitting your answers is 6:00 pm Tuesday July 14th, Toronto time. The marks allocated to each question are shown in brackets.

Questions 1, 2 and 3 are about the two-type car insurance model we analyzed last week.

1. (3 marks) Using a carefully drawn $w_gw_b$ diagram, prove that a pooling equilibrium cannot exist in this model.

**ANSWER**

See assignment 9.

2. (3 marks) What are the necessary conditions for an imperfect-information equilibrium in this model?

**ANSWER**

There cannot be a pooling equilibrium. If the insurance market exists, the only possible equilibrium is a separating equilibrium. The necessary conditions for this equilibrium are: (a) the bad driver contract is at the intersection of the 45 degree line and the $b$-line; (b) the good driver contract is at the intersection of the equilibrium bad-driver indifference curve and the $g$-line; and (c) the fraction of bad drivers is large enough that the $m$-line is either tangent to the good driver indifference or flatter than this.
3. (3 marks) (True, false and explain) In the case where \( w_0 = 100, L = 75, \pi^G = 2/3, \pi^B = 1/3, u(w) = w^{1/2} \) and half the drivers are good, there is no separating equilibrium.

**ANSWER**

False. There will be a separating equilibrium because there is a good driver indifference curve tangent to the \( m \)-line at the endowment point. This means the equilibrium good driver indifference curve lies above the \( m \)-line. To see this, calculate:

\[
\pi^M = \frac{1}{2} \pi^G + \frac{1}{2} \pi^B = \left( \frac{1}{2} \right) \left( \frac{2}{3} \right) + \left( \frac{1}{2} \right) \left( \frac{1}{3} \right) = \frac{1}{2}
\]

The absolute value of the slope of the \( m \)-line is 

\[
\text{MRS}^G_{w_g, w_b} \text{ at end. pt. } = \frac{\pi^G u'(w_0)}{(1 - \pi^G) u'(w_0 - L)} = (2) \frac{100^{-1/2}}{25^{-1/2}} = (2) \left( \frac{1}{4} \right)^{1/2} = 1.
\]

4. (3 marks) Jean is a risk-averse, expected-utility maximizer who has \( A \) dollars to invest. At the beginning of the period, she puts a fraction \( 1 - a \) of her money into a safe asset that pays a zero rate of return. And she invests fraction \( a \) of her money into a risky asset that pays \( r_1 > 0 \) in the good state and \( r_2 < 0 \) in the bad state. The government levies a proportional tax, \( 0 < t < 1 \), on her investment returns such that her net return is \((1 - t) r\) in either state. So in the good state her wealth at the end of the period would be

\[(1 - a) A + a A (1 + (1 - t) r_1) = A (1 + (1 - t) a r_1).\]

And her expected utility as a function of \( a \) and \( t \) could be written as

\[E(a, t) = \pi u (A (1 + (1 - t) a r_1)) + (1 - \pi) u (A (1 + (1 - t) a r_2)),\]

where \( \pi \) is the probability of the good state and \( u(w) \) is Jean’s Bernoulli utility of wealth function. Assume that when \( t = 0 \) Jean chooses \( 0 < a < 1 \). Does increasing the tax rate raise or lower \( a \)? Defend your answer.

**ANSWER**

We know that

\[
\text{Sign of } \frac{da}{dt} = \text{Sign of } E_{12}(a, t).
\]

Write expected utility as

\[E(a, t) = \pi u (A (1 + (1 - t) a r_1)) + (1 - \pi) u (A (1 + (1 - t) a r_2)) = \pi u (w_g) + (1 - \pi) u (w_b).
\]

Then
\[ E_1(a, t) = \pi u'(w_g)A(1 - t)r_1 + (1 - \pi)u'(w_b)A(1 - t)r_2 \]
\[ = A(1 - t)(\pi r_1 u'(w_g) + (1 - \pi)r_2 u'(w_b)) \]
\[ E_{12}(a, t) = -A(\pi r_1 u'(w_g) + (1 - \pi)r_2 u'(w_b)) + \]
\[ A(1 - t)[\pi r_1 u''(w_g)(-Aa_1) + (1 - \pi)r_2 u''(w_b)(-Aa_2)] \]

So \[ E_{12}(a, t) = -\frac{E_1(a, t)}{1 - t} + \]
\[ -aA^2(1 - t)[\pi r_1^2 u''(w_g) + (1 - \pi)r_2^2 u''(w_b)] \]

But at the optimal value of \( a \), \( E_1(a, t) = 0 \) and then the sign of \( E_{12}(a, t) \) must be positive because \( u'' < 0 \). An increase in the tax rate will induce Jean to put a larger fraction of her portfolio into the risk asset. This is an example of the moral hazard problem — offer someone insurance against bad outcomes and the person will take less care. If we had allowed the probability of an accident to be affected by the presence of insurance in the insurance model we would have obtained a similar result there.