Econ 393
Quiz 6

Instructions: You need paper (lined if possible) and a pen or a pencil to write this quiz. You may answer the questions in any order you like. You should start each question on a new page. You must write your answers; typed answers will not be accepted. When you are finished answering the questions, please order the pages so your answer to question 1 are first, then your answer to question 2, and then your answer to question 3. Then, in a single email message, send an image of each page to me at jburbidg@uwaterloo.ca. Please put Econ 393, your name and your id number in the subject line of your email. The deadline for submitting your answers is 6:00 pm Tuesday June 30th, Toronto time. The marks allocated to each question are shown in brackets.

1. (i) (2 marks) What is Paul Samuelson’s definition of a pure public good? (ii) (3 marks) Derive the Samuelson condition for efficient provision of a pure public good. Assume one public good, one private good, and two people. Denote the total supply of the private good by $Y$ and the quantity supplied of the public good by $G$; let $F(G,Y) = 0$ characterize the efficient production technologies.

**ANSWER**

(i) A Samuelson public good has the feature that if $G$ units of the public good are produced, then $G$ units of the good are consumed by each person in the economy. The good is “non-rival” — one person’s consumption of the good does not reduce the amount available for anyone else. And the good is “non-excludable” — no one can be excluded from consuming the good.

(ii) The conditions required for efficient provision of a Samuelson public good may be derived by maximizing person $B$’s utility given some fixed level of person $A$’s utility, $u_0^A$, and the economy’s efficient production technologies, $F(G,Y) = 0$. $F_1(G,Y)/F_2(G,Y)$ is the absolute value of the slope of the production possibility boundary — the marginal cost of $G$ divided by the marginal cost of $Y$.

$$\begin{align*}
\text{Max} & \quad G, y^B, Y \quad u^B(G, y^B) + \lambda (u^A(G, Y - y^B) - u_0^A) + \mu F(G,Y) \\
\text{subject to} & \quad \lambda, \mu
\end{align*}$$

The first-order conditions for $(G, y^B, Y)$, respectively, are
\[ u^B_1 + \lambda u^A_1 + \mu F_1 = 0 \quad (1) \]
\[ u^B_2 - \lambda u^A_2 = 0 \quad (2) \]
\[ \lambda u^A_2 + \mu F_2 = 0. \quad (3) \]

Rewrite (1), (2) and (3) as

\[ u^B_1 + \lambda u^A_1 = -\mu F_1 \quad (4) \]
\[ u^B_2 = \lambda u^A_2 = -\mu F_2 \quad (5) \]

Now divide each term in (4) by the corresponding term in (5) to obtain:

\[ \text{MRS}^B_{GY} + \text{MRS}^A_{GY} = \frac{\text{MC of } G}{\text{MC of } Y}. \]

2. Ann and Bruce live together. Their utility functions are:

\[ u^A(m^A, t) = m^A - (t - 20)^2 \]
\[ u^B(m^B, t) = m^B - (t - 24)^2, \]

where \( m \) is dollars spent on private goods per day and \( t \) is the temperature in the apartment in degrees Celsius. Assume Ann and Bruce each have 40 dollars to spend per day and the landlord pays any heating or cooling costs. As precisely as you can, describe the Pareto-efficient allocations in two settings: (i) (2 marks) Ann has the right to set temperature; (ii) (2 marks) Bruce has the right to set temperature.

**ANSWER**

(i) If Ann has the right to set temperature negotiations would begin at her money endowment of 40 dollars and her preferred temperature which is 20 degrees. Label this point A. Ann’s utility at this point is 40. Bruce’s favourite temperature is 24 and if Bruce gave her 16 dollars Ann would be indifferent between point A and 56 dollars and a temperature of 24. Thus, all the Pareto efficient allocations must have temperature between 20 and 24 and they are solutions to

\[ \text{MRS}^A_{mt} = \frac{1}{2(t - 20)} = \frac{1}{2(24 - t)} = \text{MRS}^B_{mt}, \]

which yields \( t = 22 \). The full set of Pareto efficient allocations for (a) are shown in Figure 1.

(ii) Here negotiations begin at Bruce’s endowment of 40 dollars and his preferred temperature of 24 degrees. Label this point B and proceed as in (i). The question re-emphasizes the point that property rights have a major impact on the set of Pareto efficient allocations. Again, see Figure 1.
3. Four identical people go out to a restaurant. Each has 72 dollars to spend on food and other goods and each has strictly convex preferences. The price of food is 12 per unit of food.

(i) (2 marks) In scenario A each pays her own bill and each buys 2 units of food. In an Econ 290 type diagram draw this equilibrium with food on the horizontal axis and dollars spent on other goods on the vertical axis.

(ii) (2 marks) In scenario B, they split the total bill evenly between them. In the same diagram you drew for part (i), draw the equilibrium for scenario B as precisely as you can.

**ANSWER**

(i) The equilibrium in scenario A has each person consuming 2 units of food and spending \( 48 = 72 - 2 \times 12 \) on other goods. With strictly convex preferences the equilibrium will be a tangency between the budget constraint and an indifference curve. See point A in Figure 2.

(ii) In scenario B everyone has the same endowment of money and the actual price of food is unchanged so the equilibrium in (ii) must lie on the same budget constraint as in (i). But the *perceived* price of food is \( 12/4 = 3 \) dollars to any one person. The equilibrium must be at a point like B in Figure 2 where the absolute value of the slope of the indifference curve is 3. Note that this must be a lower indifference curve than in (i).
Figure 1: Edgeworth rectangle for Ann and Bruce

Ann's ind. curves are red
Bruce's ind. curves are blue
At A, Ann has the right to set temp.
At B, Bruce has the right to set temp.
CD: Pareto efficient allocations for A
EF: Pareto efficient allocations for B
Figure 2

$ spent on goods other than food

(0, 72)

(2, 48)

A

|Slope| = 12

B

|Slope| = 3

(6, 0)

Units of food