Instructions: You need paper (lined if possible) and a pen or a pencil to write this quiz. You may answer the questions in any order you like. You should start each question on a new page. You must write your answers; typed answers will not be accepted. When you are finished answering the questions, please order the pages so your answer to question 1 are first, then your answer to question 2, and then your answer to question 3. Then, in a single email message, send an image of each page to me at jburbidg@uwaterloo.ca. Please put Econ 393, your name and your id number in the subject line of your email. The deadline for submitting your answers is 6:00 pm Tuesday June 23rd, Toronto time. The marks allocated to each question are shown in brackets.

1. (3 marks) The answers for assignment 6 on the web site talk about the “Dealing” row in the table for the airport-developer problem, 34.8/34.9. Let $G_A$ denote the gain in profits for the airport over what the airport earns in the private equilibrium, and $G_D$ denote the gain in profits for the developer over what the developer earns in the private equilibrium. Suppose the airport and the developer have agreed to have a mediator settle their dispute where the mediator’s objective function is

$$U(G_A, G_D) = G_A^a G_D^{1-a}, \quad 0 < a < 1.$$ 

Assume the mediator has enough information to figure out the Pareto efficient allocation and enough power to enforce it. Derive formulas for $G_A$ and $G_D$ as functions of $a$ and the numbers in the problem.

**ANSWER**

The mediator’s allocation problem is identical to an Econ 290 consumer problem where the consumer has a budget of $108 = 1008 - 900$ and the prices are 1. With a Cobb-Douglas utility function the budget shares are $a$ and $1 - a$. So

$$G_A = a \frac{108}{1} = 108a$$

$$G_D = (1 - a) \frac{108}{1} = 108(1 - a)$$
2. (2 marks for each part) Every weekday morning $\overline{N}$ commuters travel from North Guelph to South Guelph. They can either go around the town on the Hanlon or they can use Gordon Street which runs through town. The Hanlon is uncongested and takes $a$ minutes; Gordon Street takes $c + N/b$ minutes where $b$ and $c$ are constants and $N$ is the number of vehicles using the Gordon-Street route. Assume $a > c$ and $\overline{N} > 2b(a - c)$.

(i) What is total commuting time in the private equilibrium?

(ii) What is total commuting time in a socially-efficient allocation?

(iii) Assume all commuters are identical and let $w$ be the value of one minute of a commuter’s time. What toll on the Gordon-Street route would result in a Pareto-efficient equilibrium?

(iv) If Guelph decides to widen Gordon Street and reduces commute time on Gordon Street to $c + N/(2b)$ minutes would the new private equilibrium be Pareto efficient? Justify your answer.

**Answer**

(i) IF both routes are used the time taken on each route will be the same in the private equilibrium. As above let $N$ denote the number of people using the Gordon-Street route.

\[
a = c + \frac{N}{b} \text{ or } N = b(a - c).
\]

Since $\overline{N} > 2b(a - c)$ both routes are used and total commute time is $a\overline{N}$.

(ii) In a socially-efficient allocation the social planner would choose $N$ to minimize total commuting time.

\[
\text{Total time} = \left(c + \frac{N}{b}\right)N + a(\overline{N} - N).
\]

The $N$ that minimizes total commuting time is $b(a - c)/2$ and the time the Gordon-Street route will take would be

\[
c + \frac{1}{b} \frac{b(a - c)}{2} = \frac{a + c}{2}
\]

Comparing the private and socially-efficient allocations we see that in the socially-efficient allocation the time saved per commuter on the Gordon-Street route would be

\[
a - \frac{a + c}{2} = \frac{a - c}{2}.
\]

So total time saved in the socially-efficient allocation would be

\[
\frac{a - c}{2} \frac{b(a - c)}{2} = \frac{b(a - c)^2}{4}.
\]

Therefore total time taken in the socially-efficient allocation must be $a\overline{N} - b(a - c)^2/4$.

(iii) The toll must make commuters indifferent between using the Hanlon (with no toll) and using Gordon Street, paying the toll but saving $(a - c)/2$ minutes in commute time. Since $w$ is the value of a minute saved the toll should be $(a - c)w/2$.
(iv) If both routes are used, after the Gordon-Street widening, the time taken on each route will be the same in the new private equilibrium.

\[ a = c + \frac{N}{2b} \text{ or } N = 2b(a - c). \]

Since \( N > 2b(a - c) \) both routes are used and total commute time is still \( aN \). The new private equilibrium is not Pareto efficient and the widening of Gordon Street would be a waste of money.

3. Question 35.3 is about a network where there may be as many as 13 people in the network.

(i) (2 marks) How is the market in this problem different from the perfectly competitive markets you studied in Econ 391?

(ii) (3 marks) Suppose government regulations force the operator of the network to use a pricing scheme where the first \( n_1 \) customers may join the network at a price of \( p_1 \) each; the next \( n_2 \) customers may join at an individual price of \( p_2 > p_1 \) and then \( 13 - n_1 - n_2 \) customers may join at an individual price of \( p_3 > p_2 \). The rules and the pricing scheme in 35.3(c) are one example of this kind of pricing scheme. What values of \( p_i \) and \( n_i \) would maximize revenue for the network provider?

**ANSWER**

(i) On the selling side of the market the main difference is that there is one seller in question 35.3 whereas there are many sellers in the competitive markets studied in Econ 391. On the buying side of the market the main difference is not that potential customers have different valuations of the service (or good). We started this course with an Econ 391 example where there was a continuum of customers whose valuations of widgets varied from 0 to 10. What is different in 35.3 is that any customer’s valuation of the service (the network) depends on how many other people use the network. So when a person signs on to the network that creates a positive externality for everyone else who is currently using the network, and for those who might use the network.

(ii) This question asks whether, conditional on the structure of the pricing scheme in 35.3(c), the network provider has maximized its revenue. In the answer for 35.3(c) we have total revenue equal to \( 2 \times 10 + 2 \times 25 + 5 \times 45 = 295 \) with a total of 9 customers. Looking at the table, high revenue is going to result from trying to follow the cells that run from lower left to top right. The pricing scheme that maximizes revenue is \( p_1 = 13 \) where person 13 joins. Then \( p_2 = 24 \) with just two signups allowed at this price. So person 12 joins. And then with two people in the network customers 8 through 11 are willing to join. As in the answers for assignment 6, we make it hardest for ourselves by saying that customer 11 joins. And then \( p_3 = 40 \) and customers 4 through 10 join. Total revenue equals \( 13 + 2 \times 24 + 7 \times 40 = 341 \).
<table>
<thead>
<tr>
<th>Customer Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>24</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
<td>33</td>
<td>36</td>
<td>39</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
<td>32</td>
<td>36</td>
<td>40</td>
<td>44</td>
<td>48</td>
<td>52</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
<td>50</td>
<td>55</td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>54</td>
<td>60</td>
<td>66</td>
<td>72</td>
<td>78</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
<td>35</td>
<td>42</td>
<td>49</td>
<td>56</td>
<td>63</td>
<td>70</td>
<td>77</td>
<td>84</td>
<td>91</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td>40</td>
<td>48</td>
<td>56</td>
<td>64</td>
<td>72</td>
<td>80</td>
<td>88</td>
<td>96</td>
<td>104</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>45</td>
<td>54</td>
<td>63</td>
<td>72</td>
<td>81</td>
<td>90</td>
<td>99</td>
<td>108</td>
<td>117</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
<td>110</td>
<td>120</td>
<td>130</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>22</td>
<td>33</td>
<td>44</td>
<td>55</td>
<td>66</td>
<td>77</td>
<td>88</td>
<td>99</td>
<td>110</td>
<td>121</td>
<td>132</td>
<td>143</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>84</td>
<td>96</td>
<td>108</td>
<td>120</td>
<td>132</td>
<td>144</td>
<td>156</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>26</td>
<td>39</td>
<td>52</td>
<td>65</td>
<td>78</td>
<td>91</td>
<td>104</td>
<td>117</td>
<td>130</td>
<td>143</td>
<td>156</td>
<td>169</td>
</tr>
</tbody>
</table>