Instructions: You need paper (lined if possible) and a pen or a pencil to write this quiz. You may answer the questions in any order you like. You should start each question on a new page. You must write your answers; typed answers will not be accepted. When you are finished answering the questions, please order the pages so your answers to question 1 are first, and then your answers to question 2. Then, in a single email message, send an image of each page to me at jburbirdg@uwaterloo.ca. Please put Econ 393, your name and your id number in the subject line of your email. The deadline for submitting your answers is 6:00 pm Tuesday May 26th, Toronto time. The marks allocated to each question are shown in brackets.

1. (two marks for each part) Consider a world with two goods, 1, 2, many type A’s, and an equal number of type B’s. Write the endowments of each A as \((e^A_1, e^A_2)\) and each B as \((e_1 - e^A_1, e_2 - e^A_2)\), where the total endowments for an A, B pair are \((e_1, e_2)\). Assume everyone has the utility function \(u(x_1, x_2) = x_1x_2\); and the endowment point is \textit{BELOW} the diagonal of the Edgeworth rectangle (good 1 is on the horizontal axis).

   (i) Prove that, in this setting, the Pareto efficient allocations lie along the diagonal of the Edgeworth rectangle.

   (ii) Derive the equation for B’s offer curve.

   (iii) In the competitive equilibrium which good are the A’s selling and which good are they buying? Describe how you would find the equilibrium in which the A’s have a monopoly in the good they are selling.

\textbf{ANSWER}

(i) The utility function in this question is in the Cobb Douglas family, \(u(x_1, x_2) = x_1^a x_2^b, a > 0, b > 0\), and so we know upper level sets are strictly convex. It follows that the set of Pareto efficient allocations will be tangencies between the A and B indifference curves. So

\[
\text{MRS}_{12}^A = \text{MRS}_{12}^B \text{ or } \frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} = \frac{e_2 - x_2^A}{e_1 - x_1^A}
\]

Cross-multiplying and simplifying

\[
x_2^A e_1 - x_2^A x_1^A = x_1^A e_2 - x_2^A x_1^A \text{ or } \frac{x_2^A}{x_1^A} = \frac{e_2}{e_1}.
\]
This completes the proof.

(ii) $B$’s offer curve shows us the trades $B$ would be willing to make if markets were perfectly competitive. We know that with utility functions in the Cobb-Douglas family, $x_1^a x_2^b, a > 0, b > 0$, the budget share for good 1 is $a/(a + b)$. So in this example,

$$x_1^B(p_1, p_2, \text{wealth} = p_1 e_1^B + p_2 e_2^B) = \frac{1}{2} \frac{p_1 e_1^B + p_2 e_2^B}{p_1} = \frac{1}{2} \left( e_1^B + \frac{p_2}{p_1} e_2^B \right).$$

But, in this example, we know that when $B$ is in a competitive equilibrium, the MRS between goods 1 and 2 equals $p_1/p_2$, so $p_2/p_1 = x_1^B/x_2^B$. Using this in the equation above and dropping the arguments of the demand function we have

$$x_1^B = \frac{1}{2} \left( e_1^B + \frac{x_1^B x_2^B}{x_2^B} \right) \quad \text{or} \quad x_2^B = \frac{e_2^B x_1^B}{2 x_1^B - e_1^B}.$$

Note that if $x_1^B = e_1^B$, $x_2^B = e_2^B$ — $B$’s offer curve must pass through $B$’s endowment point.

(iii) Since we have assumed the endowment point is below the diagonal of the Edgeworth rectangle, and, in this example, the diagonal comprises the Pareto efficient allocations, the $A$s will be selling good 1 and buying good 2. In the equilibrium in which the $A$s have a monopoly in the sale of good 1 and all units of good 1 are sold at the same price the $A$s will maximize their utility subject to $B$’s offer curve, which is stated in part (ii) of this question.

2. The Hamilton Tiger Cats have a football stadium that can seat 30,000 people. For a regular home game the club maximizes profits by charging 30 dollars per ticket and selling 18 thousand tickets. Assume all seats are equally desirable, the marginal cost of seating another fan at the game is zero and the market demand for tickets sold in Hamilton is linear.

(i) (4 marks) What is the equation for the market demand for tickets to a Tiger Cat game?

(ii) (6 marks) Suppose the Tiger Cats have a playoff game at home against the Montreal Alouettes next November. Further, assume the market demand in Hamilton is the same for playoff game as it is for a regular season game, the Tiger Cats can sell an unlimited number of seats in Montreal at 30 dollars per seat, and tickets purchased in Montreal cannot be resold in Hamilton. To maximize Tiger Cat profits: how many tickets should be sold in Hamilton, how many tickets should be sold in Montreal, and what price should be charged to fans in Hamilton?

(i) The market demand in Hamilton is linear so write the (inverse) market demand as $p = a - bQ$, where $p$ is the price in Hamilton, $Q$ is seats sold to Hamilton fans and $a$ and $b$ are positive parameters. For a regular home game there is no price discrimination so total revenue is $pQ = aQ - bQ^2$ and therefore marginal revenue is $a - 2bQ$. The question says marginal cost is zero and the Tiger Cats act to maximize profits. Thus $a - 2bQ = 0$ or $Q = a/(2b)$. Using the inverse market demand, $p = a/2$. Since $p = 30$ and $Q = 18000$, $a = 60$ and $b = 1/600$. So the inverse market demand is $p = 60 - Q/600$ and the market demand is $Q = 36000 - 600p$. 

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(ii) To maximize profits on the playoff game the Tiger Cats should make marginal revenue in Hamilton equal marginal revenue in Montreal. Continuing to denote tickets sold in Hamilton by $Q$ we have

$$60 - \frac{2Q}{600} = 30 \text{ or } Q = 9000.$$ 

Since the stadium capacity is 30000, 21000 tickets should be sold in Montreal. The price charged in Hamilton should be $60 - 9000/600 = 45$ dollars per seat.